Physics 8.01 T – Section L05

Solutions to Quiz 3

Part (a)

The first part of the question clearly just involves circular motion. If I draw a free-body diagram for the ball at the top of the circle, I have a formula that tells me the total force this ball should experience. However, this formula involves the velocity of the ball at the top of the circle, so I'll need to find that.

That said, my strategy will be as follows:

1. I'll find the velocity of the ball at the top of the circle using conservation of energy.
2. I'll use this to find the total force the ball should be experiencing, using the equations of circular motion.
3. I'll draw a free-body diagram for the ball there.
4. I'll use that to find the tension, T.

Part (b)

Step 1 The easiest way to find the speed of the ball at the top of the circle is using conservation of energy. Let v_0 be the original speed of the ball, and v_t be the speed at the top of the circle. The ball ends up a height $2l$ above its original stating point. So the energy balance is as follows

	Initially	Top of the circle
Kinetic energy	$\frac{1}{2}mv_0^2$	$\frac{1}{2}mv_t^2$
Potential energy	0	$2mgl$
Total	$\frac{1}{2}mv_0^2$	$\frac{1}{2}mv_t^2 + 2mgl$

There are no other forces, and so the total energy must be the same originally and at the top of the circle. So

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$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_t^2 + 2mgl$$
$$mv_0^2 = mv_t^2 + 4mgl$$
$$mv_t^2 = mv_0^2 - 4mgl$$
$$v_t^2 = v_0^2 - 4gl$$
$$\boxed{v_t = \sqrt{v_0^2 - 4gl}}$$

Step 2 We know, from the equations of circular motion, that

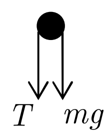
$$F = \frac{mv_t^2}{l}$$

Feeding in the v_t we found above

$$\boxed{F = \frac{m\left(v_0^2 - 4gl\right)}{l}}$$

And this force must point *downwards*, towards the centre of the circle.

Step 3 A free-body diagram for the particle at the top of the circle is



Step 3 And so

$$F = T + mg$$
$$\frac{m\left(v_0^2 - 4gl\right)}{l} = T + mg$$
$$T = \frac{m\left(v_0^2 - 4gl\right)}{l} - mg$$
$$T = \frac{mv_0^2}{l} - 4mg - mg$$
$$\boxed{T = \frac{mv_0^2}{l} - 5mg}$$

Part (c)

This is simply a case of projectile motion in 2-dimensions, which we covered earlier this year. At the top of the circle, the velocity is purely horizontal, and so our strategy is simply

1. Find the time taken for the particle to drop the height *2l* vertically, given that it started with 0 vertical velocity.
2. Work out how far it would have travelled horizontally in that time.

Step 1 Original velocity is 0, acceleration is g and distance travelled is *2l*, and so the time taken is

$$x = v_0 t + \frac{1}{2} a t^2$$
$$2l = \frac{1}{2} g t^2$$
$$4l = g t^2$$
$$\boxed{t = 2\sqrt{\frac{l}{g}}}$$

Step 2 The original horizontal velocity (from part A) was

$$v_t = \sqrt{v_0^2 - 4gl}$$

And so the total distance travelled is

$$d = t v_t$$
$$d = 2\sqrt{\frac{l}{g}}\sqrt{v_0^2 - 4gl}$$
$$\boxed{d = 2\sqrt{\frac{\left(v_0^2 - 4gl\right) l}{g}}}$$

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