Two Echelon Distribution Systems with Random Demands and Storage Constraints

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Abstract

We consider a general two-echelon distribution system consisting of a *depot* and multiple sales outlets, henceforth referred to as *retailers*, which face random demands for a given item. The replenishment process consists of two stages: the depot procures the item from an outside supplier, while the retailers' inventories are replenished by shipments from the depot. Both of the replenishment stages are associated with a given facility-specific leadtime. The depot as well as the retailers face a limited inventory capacity. Inventories are reviewed and orders are placed on a periodic basis. When a retailer runs out of stock, unmet demand is backlogged.

We propose a new approach to the above class of dynamic programming models based on Lagrangian relaxation. Every choice of the vector of Lagrange multipliers generates a lower bound via the solution of a *single* DP with a one-dimensional state-space. The best such bound is obtained by maximizing over the vector of multipliers. The strategy that is optimal for this (maximal) lower bound DP employs an (s, S) *ordering* policy (generally, with time-dependent policy parameters). To arrive at an upper bound and an implementable heuristic, this (s, S) policy is paired with one of several possible *allocation* policies that allocate the system-wide inventory across the different facilities.

We report on an extensive numerical study with close to 14,000 instances which evaluates the accuracy of the lower bound and the optimality gap of the various heuristic policies. The study reveals that the lower bound and the heuristic strategy that is constructed on its basis perform exceedingly well, almost across the entire parameter spectrum, including instances where demands are rather volatile or the average cycle time between consecutive orders is relatively large. The exception arises when storage at the depot is as expensive as at the retailer level and the retailers have large storage capacities.

1 Introduction

We consider a general two-echelon distribution system consisting of a *depot* and multiple sales outlets, henceforth referred to as *retailers*, which face random demands for a given item. The replenishment process consists of two stages: the depot procures the item from an outside supplier, while the retailers' inventories are replenished by shipments from the depot. Both of the replenishment stages are associated with a given facility-specific leadtime. The depot as well as the retailers face a limited inventory capacity. Inventories are reviewed and orders are placed on a periodic basis. When a retailer runs out of stock, unmet demand is backlogged.

In such systems, the challenge is to find an optimal trade-off among the following four cost components: (i) costs associated with the orders placed with the external supplier, typically reflecting economies of scale, (ii) shipment costs for transfers from the depot to the retailers, (iii) carrying costs associated with the depot's and retailers' inventories at the end of each period, and (iv) backlogging costs at the retailers, again a function of the backlog size there at the end of each period. The objective is to minimize the expected discounted or average system-wide costs over a finite or infinite planning horizon.

For most of this paper, we make the following *standard* structural assumptions about these cost components: (i) orders costs have a fixed and a variable cost component, (ii) the internal shipment costs are linear in the shipment volumes, at facility-specific cost rates, (iii) the carrying cost at each facility in each period is a convex function of its ending inventory level, (iv) the backlogging costs at each retailer, at the end of each period, is also a convex function of its ending backlog size.

However, our fundamental results continue to hold under *fully general* shapes of the various cost functions, as long as the internal shipment costs, see (ii), are linear. Starting from an exact, albeit intractable, dynamic program (DP) formulation of the problem, we show that the solution of the Lagrangian dual to this problem requires only the solution of a DP with a *one-dimensional* state space, for any given choice of the Lagrange multipliers. Moreover, the optimal strategy of this lower bound DP generates a feasible order policy that can be paired, effectively, with one of several heuristic shipment/allocation policies determining, in each period, how much of the depot's inventory is to be withdrawn, and how it should be allocated among the various retailers. The expected cost value of this order and allocation policy is, of course, an *upper bound* for the optimal cost value. We show that the expected cost of this feasible policy is very close to the lower bound, establishing both that the policy is close to optimal, and that the Lagrange dual yields a tight lower bound.

Under the above structural assumptions the optimal order strategy in the lower bound DP is of a simple (s, S) type, typically with time-dependent policy parameters s and S, and execution of the above heuristic shipment/allocation policy involves the solution of a non-linear program in each period. Under the convexity assumptions (iii) and (iv), the non-linear program is a convex program with a special structure, allowing for very efficient solution procedures.

The above problem class is clearly of central and pervasive importance. The *uncapacitated* version of the problem, i.e. without capacity constraints on the inventory that can be stored, was first considered under the structural assumptions in (i)–(iv) in Clark and Scarf (1960), generally viewed as one of the seminal papers in the field of inventory theory. The authors focused, for most of the paper, on the special case where all demands are met at a *single* retail location. In that case, it is possible to show that the *optimal* strategy can be found by solving a *nested* pair of dynamic programs (DP), each with a one-dimensional state space. The dynamic program associated with the depot employs so-called *induced penalty functions* derived from the value functions of the associated retailer's dynamic program. Moreover, the optimal replenishment strategy in this nested pair of DPs has a very simple structure: the retailer follows a modified base-stock policy, i.e., at the beginning of every period, she brings her inventory position *as close as possible* to a given *base-stock level*. The depot places orders in accordance with an (s, S) policy, i.e. he elevates the system-wide inventory position (otherwise known as the echelon inventory position) to a level S if and only if it has dropped to or below a re-order level s. (All policy parameters may be time-dependent).

However, the problem becomes very complex when the depot serves *multiple* retailers; the most concise known dynamic programming formulation involves a DP with a state space of a dimension given by the sum of the number of retail locations J and L, the first-stage lead-time (plus one). In particular, when there are two or more retailers, the above technique of decomposing the overall DP into a series of nested DPs breaks down. Traditional (brick-and-mortar) chains have between scores and thousands of stores; .e.g. Wall-mart has 5000 stores and wholesale clubs in the US alone. Even internet retailers typically serve consumers from many fulfillment centers: Amazon, for example, had, in 2017, a network of 214 such centers. Solution of the exact DP is, thus, impossible for virtually all real-life applications. In addition, even if this exact DP could be solved, the resulting optimal strategy would be so complex as to defy implementation.

Clark and Scarf (1960), themselves, proposed a first heuristic approach to this problem, based on the so-called *balance assumption*, i.e. the assumption that the distribution of inventories among the retailers is the one which minimizes future expected costs. In other words, retailer inventories, at the beginning of *each* period are *perfectly* balanced. The balance assumption results in a *lower* bound approximation, and one that decomposes into J separate retailer problems along with a DP for the depot which derives its cost terms from the value functions of the retailers' DP. The method is similar to the one described above for the single retailer case. Clark and Scarf (1960) did not carry out a numerical study to evaluate the accuracy of the proposed lower bound or the optimality gap of the policies generated under their approach.

Federgruen and Zipkin (1984b) proposed a different lower bound approximation, however under various restrictions, in particular, that no inventories may be kept at the depot (i.e. its inventory capacity is zero), while the capacity of the retailers is infinite. Their approximation is based on a relaxation of the non-negativity constraints for the shipments from the depot to the retailers; after this relaxation, the exact DP can be reduced to one with a one-dimensional state-space.

As a variant of the relaxation approach in Federgruen and Zipkin (1984b), Kunnumkal and Topaloglu (2008) allow for the depot to store inventories, but ignore any fixed components in the replenishment costs. They proposed a *Lagrangian relaxation* approach in which the same non-negativity constraints are eliminated from the constraint set, and are then incorporated into each period's cost function via Lagrange penalty terms. Following the single-retailer approach of Clark and Scarf (1960), they then show that the Lagrangian relaxed problem can be decomposed into J independent retailer DPs, followed by a depot DP which uses all of the value functions from the retailer DPs when constructing its one-step expected value functions. A lower bound is obtained for any combination of Lagrange multipliers, and as in standard Lagrangian relaxations for mathematical programs, the *best* such lower bound can be found by embedding the sequence of nested DPs in a maximization procedure over all non-negative vectors of the Lagrange multipliers.

Most recently, Marklund and Rosling (2012) developed a new approach to this problem, but only for infinite-horizon stationary models, and making an upfront restriction to so-called (m, v) policies. Under an (m, v) policy, an order is placed with the supplier every m periods, so as to bring the system-wide inventory position up to a target level v. The authors make the heuristic assumption that order sizes are large enough "for the decisions in each order cycle to be considered in isolation" (p. 94). They show that this myopic approximation is asymptotically optimal when the number of retailers goes to infinity, under additional conditions. However, the solution of a high-dimensional, and hence intractable, dynamic program is needed in order to to determine how the system-wide inventory at the beginning of a cycle is optimally distributed during the cycle. The authors formulate the problem as an m-step stochastic program and apply *two types* of relaxations to solve this (single-cycle) problem. The result is a lower bound for the optimal system-wide cost in a single cycle under a given (m, v)-order policy; the strategy optimizing this lower bound may be amended to a feasible heuristic distribution strategy in several ways.

We report on an extensive numerical study with close to 14,000 instances which evaluates the accuracy of the lower bound and the optimality gap of the various heuristic policies. The study reveals that the lower bound DP and the heuristic strategy that is constructed on its basis perform excellently, almost across the entire parameter spectrum, including instances where demands are rather volatile or the average cycle time between consecutive orders is relatively large. The exception arises when storage at the depot is as expensive as at the retailer level and the retailers have large storage capacities.

The remainder of this paper is arranged as follows. In section 2, we review the related literature. The model and notation are introduced in section 3. In section 4, we develop an exact DP formulation. For finite horizon models, section 5 develops our Lagrangian-relaxation based lower bound approximate dynamic program, while section 6 discusses various heuristic allocation policies to complement the ordering strategy which is optimal in the approximate DP associated with the optimal Lagrange multipliers. Section 7 covers infinite horizon models, both under the present value and the long-run average cost condition. Our numerical study is discussed in section 8. Section 9 completes this paper with various conclusions and future generalizations of our results.

2 Literature Review

As explained in the introduction, Clark and Scarf (1960), Federgruen and Zipkin (1984b), Kunnumkal and Topaloglu (2008), and Marklund and Rosling (2012) considered three fundamental approaches for the control of one-depot, multi-retailer systems with stochastic demands and order and shipment leadtimes: Clark and Scarf (1960) developed a tractable lower bound based on the 'balance assumption', and Federgruen and Zipkin (1984b) showed, under various restrictions discussed in the introduction, how the straightforward relaxation of the non-negativity constraints for the shipment quantities results in a lower bound which can be reduced to a standard single location inventory DP. The authors also showed how a heuristic policy can be constructed based on the strategy that is optimal in this lower bound DP. Ignoring fixed order costs, Kunnumkal and Topaloglu (2008) extended this approach by replacing the 'simple' relaxation of the non-negativity constraints by a Lagrangian relaxation approach, also allowing for inventories to be stored at the depot and general demand distributions and cost parameters.

Several other papers have built on and complemented these methodological approaches. Eppen and Schrage (1981) considered the special case of a distribution system without central inventories and with fully identical retailers and Normal stationary demands. Most importantly, the authors make an upfront restriction to base-stock ordering policies from the external supplier. Based on the 'balance assumption' introduced in Clark and Scarf (1960), the authors derive a closed form lower bound for the long-run average costs in the system under any given base-stock level, as well as a base-stock level that minimizes this closed-form lower-bound. Jackson (1988) extended the Eppen and Schrage (1981) results to allow for central inventories. Axsäter, Marklund, and Silver (2002) employ the balance assumptions for systems in which all supplier orders must be multiples of an exogenously given batch size. The literature that revolves around the 'balance assumption' was reviewed by Axsäter (2003). We refer to Doğru (2005) and Doğru, De Kok, and Van Houtum (2004) for extensive numerical studies evaluating the accuracy of the 'balance assumption'.

The relaxation approach in Federgruen and Zipkin (1984b) was complemented by two additional papers by the authors. For an infinite-horizon model with stationary parameters, Federgruen and Zipkin (1984a) showed that the relaxation approach may, sometimes, result in superior policies when the orders from the external supplier are, a priori, restricted to a specific class of ordering policies acting on the system-wide inventory position – for example, (s, S) policies, or the above-mentioned (m, v) policies. This paper also discusses alternative allocation policies beyond the myopic or greedy allocations discussed in the introduction. As in Federgruen and Zipkin (1984b), the paper confines itself to systems without depot inventories and Normal demands. This part of the literature was reviewed by Federgruen (1993).

Federgruen and Zipkin (1984c) consider the relaxation approach for systems *with* depot inventories. The authors show that a lower bound can be obtained by solving a pair of *nested* DPs, similar to what is optimal in the single retailer case of Clark and Scarf (1960). No numerical studies have been reported to validate this approximation approach. Gallego, Özer, and Zipkin (2007) employ the relaxation approach in a continuous review variant of the model in which the retailers face *independent* Poisson demand processes and each facility employs an echelon inventory position based base-stock policy.

As explained below, the Lagrangian relaxation approach of Kunnumkal and Topaloglu (2008) is very time consuming. As a consequence, instead of conducting a full subgradient based optimization over the space of all Lagrange multiplier vectors, Kunnumkal and Topaloglu (2011) suggest several heuristics to identify a *single* vector of Lagrange multipliers.

Several papers have incorporated capacity constraints in stochastic inventory models. Among those, almost all consider bounds on individual *order quantities*. Under this type of capacity constraint, even the single facility problem becomes fundamentally more complex both structurally and computationally. Shaoxiang and Lambrecht (1996) demonstrate that the optimal policy admits an $X-Y$ band structure, with $X < Y$. If the inventory position is below X, order the full capacity. If it is above Y, order nothing. However, if the inventory position is located between the two thresholds, then the ordering policy is more complicated. Parker and Kapuscinski (2004), Chao and Zhou (2009), and Janakiraman and Muckstadt (2009) propose heuristics for a *capacitated serial* system in which each facility distributes to a single successor facility. To our knowledge, there is no literature on stochastic one-depot *multiple-retailer* systems with order capacity constraints.

Finally, there is also a paucity of multi-location models with inventory capacity constraints. However, the importance of such constraints was recognized in the early contributions to inventory theory, spearheaded by the late Veinott, one of the founding fathers of the field of inventory theory to whose memory this article is dedicated. Veinott (1965), Bessler and Veinott (1966) and Ignall and Veinott (1969) consider such inventory bounds in the special case of our system where the order lead time equals zero and no inventories may be kept at the depot, or no depot exists, to begin with. Moreover, no economies of scale in the cost structure are considered i.e. all ordering costs are assumed to be linear in the order quantities. In this special setting, *myopic* policies are in fact optimal, under specific additional assumptions.

We conclude this Section with a comparison of our approach and contributions versus those in Federgruen and Zipkin (1984b), Kunnumkal and Topaloglu (2008), and Marklund and Rosling (2012).

First, compared with Federgruen and Zipkin (1984b), these are the points of differentiation. (1) Federgruen and Zipkin (1984b) assume that no inventory can be carried at the depot. In other words, the depot functions solely as a coordinating facility which enables postponement of retailer specific allocations until the supplier order arrives at this facility. (Indeed, this special case of the general model applies even when there is no physical depot, and the latter is a virtual coordinator only). In contrast, depending on the choice of the depot capacity level, we allow for centralized inventories at the depot or the absence thereof. (2) We impose capacity constraints on the amount of inventory that can be carried at any one of the retailers or the depot. Indeed, physical storage limitations are an important factor in many distribution systems. Storage space is often sparse in brick-and-mortar stores and the same applies to fulfillment centers of online retailers. Since, in general, there is a positive shipment leadtime between the allocation of a shipment quantity to a retailer and its receipt there, the inventory level at the start of the period in which the shipment arrives is random. To guarantee that there is never any overflow at any of the facilities, one would need to focus on a 'worst case' scenario, in which *no* demand is observed at the retailer during the entire shipment leadtime. This would result in poor use of the inventory space. We therefore formulate the retailer's inventory capacity constraints as chance constraints limiting the probability of an overflow of items in any given period at any of the facilities to an acceptable threshold value. (The worst case planning approach is easily recovered by setting the overflow probability to zero). (3) Federgruen and Zipkin (1984b) require all retailers' single period demands to be normally distributed[1]. Our approach allows for arbitrary distributions. (4) The approximate DP in Federgruen and Zipkin (1984b) is guaranteed to generate a *lower bound* only when the backlogging and storage cost rates are *identical* across all retailers. Under general cost rates, Federgruen and Zipkin (1984b)'s approximate DP involves additional approximation steps, beyond the above mentioned relaxations. Our approach applies to *arbitrary* cost parameters and results in a lower bound throughout. (5) We allow for variable retailer-specific shipment costs from the depot to the retailers.

When comparing our approach to that in Kunnumkal and Topaloglu (2008), we note: (1) Kunnumkal and Topaloglu allow for depot inventories, general demand distributions and general cost parameters as do we, but they continue to assume that arbitrarily large inventories may be carried at any of the facilities, and that only *variable* and *no* fixed replenishment costs prevail. We incorporate (i) the above chance constraints for the inventory carried by each facility in each period and (ii) fixed ordering costs. (2) The Kunnumkal and Topaloglu (2008) methodology is considerably more time-consuming. For any *given* vector of Lagrange multipliers, it requires the solution of $(J+1)$ nested dynamic programs, i.e., one dynamic program for each retailer and one additional dynamic program for the depot which uses the value functions of the retailer dynamic programs as inputs. In contrast, our approach requires the solution of a *single* dynamic program. Both the size of the state space and that of its action sets is comparable across these various dynamic programs. (The retailer

[1] In some special cases, they allow all demand variables pertaining to any given period t to have a CDF which is unique up to a centralization and scaling parameter, thus allowing for exponential and gamma distributions with a common scale parameter.

dynamic programs are, however, simpler to solve than the depot-based dynamic program.) Recall that in many applications, J may be in the hundreds of thousands. This is all the more important since the single dynamic program – or the $(J+1)$ dynamic programs in Kunnumkal and Topaloglu's approach – need to be repeated hundreds of times as part of a steepest ascent method to solve the Lagrangian dual within a given precision to find the vector of Lagrange multipliers maximizing the lower bound. (Kunnumkal and Topaloglu (2008) report that the required CPU times are over 600 times as large as those incurred when solving a *single* sequence of nested DPs.) The fact that, in the Kunnumkal and Topaloglu (2008) approach, the evaluation of a *single* lower bound, for a single vector of Lagrange multipliers, is very time consuming motivated the authors, in Kunnumkal and Topaloglu (2011), to suggest several simple heuristics for the choice of this multiplier vector, as opposed to a full subgradient based optimization over the full space of multiplier vectors. (3) A large part of the computational effort involved in both our and Kunnumkal and Topaloglu's approach stems from the solution of a non-linear allocation problem for *each state and action value* in each of the T periods of the planning horizon. We show that our structure enables various simplifications that are unavailable for the allocation problem in Kunnumkal and Topaloglu (2008) (which employs the value functions for the nested dynamic programs). (4) Last, but not least, we design a significantly improved heuristic strategy and associated upper bound for the optimal cost value by introducing a new allocation problem. This problem allocates new orders and central inventories so as to optimize expected costs over a time window of a given length κ periods, as opposed to the 'greedy' procedure in Federgruen and Zipkin (1984b) and Kunnumkal and Topaloglu (2008) which allocates to optimize costs only in the *first* period in which the allocations arrive at their destinations (the recommended choice of the allocation planning window κ is determined as a function of various system parameters). Usage of this κ-allocation problem is practically beneficial when system-wide orders enter the system only intermittently, as is the case when orders are associated with significant fixed costs; this challenge is, therefore, absent in Kunnumkal and Topaloglu (2008). An example, at the end of our numerical study, explains why the new allocation policy results in significant cost savings.

Finally, the following are the primary differences between the approach in Marklund and Rosling (2012) versus ours. (1) Marklund and Rosling's methodology is restricted to finite horizon models with stationary parameters and distributions, whereas ours handles general non-stationarities (2) Marklund and Rosling make an upfront restriction to a class of order policies, so-called (m, v) policies, and develop a lower bound for the optimal costs in a given cycle under a given starting system-wide inventory. We develop a lower bound for the optimal cost under any feasible strategy and compare it with specific feasible strategies. (3) Marklund and Rosling assume that arbitrarily large inventories may be carried at any of the retailers; we include inventory capacity limits enforced via chance constraints for overflow events.

3 Model and Notation

The general description of our system is provided in the introduction. We consider a periodic review system with a finite or infinite planning horizon of $T \leq \infty$ periods. Our objective is to minimize expected discounted aggregate costs in the system, or the average cost per period. As in most standard inventory models, we assume that all stockouts at the retailers are fully backlogged, and that in any given period, the carrying and backlogging costs at each retailer are assessed as a convex function of the end-of-period inventory and backlog size, respectively. The same applies to the depot, except that no backlogs occur there.

We assume that storage space may be limited, at each of the system's facilities. However, we model the resulting capacity constraints allowing for a degree of flexibility as follows: when an allocation is made to a retailer, one needs to ensure that space is available there when the allocation reaches the retailer, i.e., *a shipment leadtime later*. One could, in principle, require that the current inventory position of the retailer plus the new allocation stay below the inventory capacity a shipment leadtime later, since this represents an upper bound for the then prevailing inventory level. However, that approach is extremely conservative, leaving a significant part of the storage space unused, in particular when the shipment leadtime is significant. Instead, we specify a chance constraint, for every retailer in every period, to ensure that the likelihood of an overflow, a shipment leadtime later, is bounded by a given probability. In almost all practical applications, an overflow can be accommodated by renting or utilizing nearby external storage space on an ad-hoc basis, albeit potentially at a high additional cost[2]. Note that, in general, a standard capacity constraint can be modeled by setting the maximum overflow probability equal to *zero*, which ensures that the storage space is adequate under *all* demand scenarios. [3]

[2]To the extent the cost of overflow storage requirements is easily predictable, one may incorporate this information by adjusting the shape of the retailer's carrying cost function for inventory levels beyond the maximal capacity, all the while preserving its convexity structure.

[3]However, when demand distributions are chosen with a support that fails to be bounded from below, for example Normal distributions, a rigid specification of the storage capacity constraints results in an infeasible dynamic program.

In a similar vein, we model any storage capacity constraint at the depot via a non-linear carrying cost function the slope of which increases greatly at the 'normal' capacity bound. (Specification via a simple hard capacity constraint may result in an infeasible DP).

The sequence of events in each period is as follows. (1) Deliveries to the depot and retailers are received. (2) Decisions are made as to how inventory at the depot should be apportioned, whether the depot should place a new order, and if so of what size. (3) Demand is observed. (4) Holding and backorder costs are assessed at the retailers, as well as carrying costs at the depot, given the remaining inventory levels at these various facilities.

Before we define our notation in detail, we note the following general conventions.

- Stores are indexed by $j = 1, \cdots, J$, positioned in the *subscript* of the variable in question.
- Lowercase Latin letters denote costs and inventories at the stores, whereas uppercase Latin letters will denote costs and inventories at the depot.
- We index time by $t = 1, \cdots, T$, positioned in parentheses in the *subscript* of the variable in question.

Bearing this in mind, we use the following notation.

Infrastructure data :

- J: numbers of retailers, indexed by j.
- L: leadtime for orders to the central depot.
- ℓ_j: leadtime for allocations to retailer j from the central depot.

Capacity data :

- $\chi_{D,(t)}$: storage capacity of the depot in period t.
- $\chi_{j,(t)}$: storage capacity of retailer j in period t.
- α_j: the acceptable probability of an overflow at retailer j in any given period.

Costs :

- $K_{(t)}$: the fixed cost to place an order to the depot, in period t.

Under such demand distributions, it is essential to specify the acceptable overflow probability as a strictly positive number.

- $c_{(t)}$: the variable procurement rate for the depot in period t.
- $\gamma_{j,(t)}$: the variable shipment cost rate for retailer j in period t.
- $p_{j,(t)}(\cdot)$: the backlogging costs for retailer j at the end of period t, as a function of the total backlog size.
- $h_{j,(t)}(\cdot)$: the inventory carrying costs for retailer j at the end of period t, as a function of the total inventory level.
- $H_{(t)}(\cdot)$: the inventory carrying costs at the depot at the end of period t, as a function of the total inventory level.

Demand : The one-period demand at retailer j in period t is represented by $u_{j,(t)}$. We assume that demands are uncorrelated across periods, but not necessarily across retailers. More specifically, we assume that the vector of demand variables $(u_{1,(t)}, \cdots, u_{j,(t)})$ has a general, continuous joint distribution. The assumption of intertemporal independence of the demands is standard in virtually all inventory models and avoids keeping track of past demand realizations as part of the state description.

We are now ready to develop an exact DP formulation of the problem.

State of the system : The following variables fully determine the state of the system at the start of any given time period t:

- $x_{j,(t)}$: the total inventory position at retailer j at the beginning of period t. This includes the total inventory on hand at the retailer at the start of the period, as well as all allocations currently in the pipeline, on their way to the retailer.
- $X_{(t)}$: inventory at the depot at the start of period t, before receipt of any order placed L periods before.
- $W_{(\tau)}$: outstanding order to the depot placed in period τ, for delivery in period $\tau + L$. At the start of period t, we shall only require these variables for time periods $\tau = t - L, \cdots, t-1$.

Thus, the $(J + L + 1)$-tuple $(x_{1,(t)}, \cdots, x_{J,(t)}, X_{(t)}, W_{(t-L)}, \cdots, W_{(t-1)})$ serves as the state of the system at the beginning of period t.

Actions taken in period t **:** In period t, the following decisions need to be made. Note that each decision affects a state variable in the next period; we denote each decision by the same symbol as the corresponding state variable, but with the addition of a bar.

- $\bar{x}_{j,(t)}$: the total amount of inventory allocated to retailer j in period t. This quantity is the sum of the inventory position at retailer j at the start of period t *plus* the shipment allocated to retailer j in this period.
- $\bar{X}_{(t)}$: how much inventory remains in the depot in period t after receipt of any incoming orders and after allocations to the retailers.
- $\bar{W}_{(t)}$: the size of the order to place from the external supplier in period t.

Finally, we let $\beta \leq 1$ denote the one-period discount factor with which we discount future costs. To formulate the problem, we use the following general conventions.

- A + sign in the subscript implies summation over all retailers. For example, $x_{+,(t)} = \sum_{j=1}^{J} x_{j,(t)}$.
- A bold symbol with a missing retailer subscript denotes a vector over all retailers. For example, $\boldsymbol{x}_{(t)} \equiv (x_{1,(t)}, \cdots, x_{J,(t)})$.

  $\boldsymbol{W}$ is used to represent the vector of orders placed in the previous L periods, i.e.,

$$\boldsymbol{W}_{(t)} \equiv (W_{(t-L)}, \cdots, W_{(t-1)})$$

- When considering demand variables, a hat indicates demand over the next L time periods, and when considering demand variables pertaining to retailer j, a dot indicates demand over the next ℓ_j time periods, and a tilde demand over the next $\ell_j + 1$ time periods.

  For example, $\hat{u}_{j,(t)} = \sum_{\tau=t}^{t+L-1} u_{j,(\tau)}$, $\dot{u}_{j,(t)} = \sum_{\tau=t}^{t+\ell_j-1} u_{j,(\tau)}$, and $\tilde{u}_{j,(t)} = \sum_{\tau=t}^{t+\ell_j} u_{j,(\tau)}$.

4 An Exact Dynamic Programming Formulation

In this section, we derive an exact dynamic programming formulation, which uses the $(J+L+1)$-tuple $(x_{1,(t)}, \cdots, x_{J,(t)}, X_{(t)}, W_{(t-L)}, \cdots, W_{(t-1)})$ as the state of the system at the start of a given period t. This state description is maximally concise. As discussed in the introduction, the exact DP is, by itself, of very limited value, since in all but the most trivial applications, its $(J+L+1)$-dimensional state space precludes computational tractability. However, the exact DP provides the starting point for the relaxation approach proposed in this paper.

Since any stockouts at a retailer are fully backlogged, a retailer's inventory level at the end of period $(t+\ell_j)$ equals its inventory position $\bar{x}_{j,(t)}$ at the beginning of period t *after* inclusion of any allocations made by the depot, minus the cumulative demand $\tilde{u}_{j,(t)}$ that occurs in the interval $[t, t+\ell_j]$. This simple identity allows us to express the expected carrying and backlogging costs, at the end of period $t+\ell_j$, as a function of $\bar{x}_{j,(t)}$ only; we call this function $Q_{j,(t)}$, where

$$Q_{j,(t)}(\bar{x}_{j,(t)}) \equiv \beta^{\ell_j}\mathbb{E}\left\{h_{j,(t+\ell_j)}(\bar{x}_{j,(t)} - \tilde{u}_{j,(t)}) + p_{j,(t+\ell_j)}(\bar{x}_{j,(t)} - \tilde{u}_{j,(t)})\right\}$$

Similarly, the starting inventory level at period $t+\ell_j$ is given by $\{\bar{x}_{j,(t)} - \dot{u}_{j,(t)}\}$. Thus, the chance constraint that limits the likelihood of an overflow of inventory at the start of period $t+\ell_j$ to α_j can be stated as

$$\begin{aligned}(1-\alpha_j) \leq \mathbb{P}\left[\bar{x}_{j,(t)} - \dot{u}_{j,(t)} \leq \chi_{j,(t+\ell_j)}\right] &= 1 - \dot{F}_{j,(t)}(\bar{x}_{j,(t)} - \chi_{j,(t+\ell_j)}) \\ \Leftrightarrow \dot{F}_{j,(t)}(\bar{x}_{j,(t)} - \chi_{j,(t+\ell_j)}) &\leq \alpha \\ \Leftrightarrow \bar{x}_{j,(t)} \leq \chi^{[\alpha_j]}_{j,(t+\ell_j)} &\equiv \chi_{j,(t+\ell_j)} + \dot{F}^{-1}_{j,(t)}(\alpha_j) \end{aligned} \quad (1)$$

where $\dot{F}_{j,(t)}$ denotes the CDF of the aggregate demand $\dot{u}_{j,(t)}$ and $\dot{F}^{-1}_{j,(t)}$ its inverse.

Thus, the *inventory level* chance constraints may be formulated as an equivalent simple upper bound on the *inventory position*, one shipment leadtime earlier. We shall refer to $\chi^{[\alpha_j]}_{j,(t+\ell_j)}$ as the *extended* capacity at retailer j in period t.

The complexity of evaluating the functions $Q_{j,(t)}(\cdot)$ and the extended capacity values $\chi^{[\alpha_j]}_{j,(t+\ell_j)}$ depends on the ease with which the CDFs of the convolutions $\tilde{u}_{j,(t)}$ and $\dot{u}_{j,(t)}$ can be determined, as well as the complexity of the functions $h_{j,(t)}(\cdot)$ and $p_{j,(t)}(\cdot)$. A particularly simple setting arises when one-period demands $u_{j,(t)}$ have a $N(\mu_{j,(t)}, \sigma^2_{j,(t)})$ distribution, and the functions $h_{j,(t)}(\cdot)$ and $p_{j,(t)}(\cdot)$ are linear. In this case, it is well-known that the function $Q_{j,(t)}(\cdot)$ admits a closed form expression involving the PDF and CDF of the standard Normal; similarly,

$$\chi^{[\alpha_j]}_{j,(t+\ell_j)} = \chi_{j,(t+\ell_j)} + \sum_{\tau=t}^{t+\ell_j-1} \mu_{j,(\tau)} + \Phi^{-1}(\alpha)\sqrt{\sum_{\tau=t}^{t+\ell_j-1} \sigma^2_{j,(\tau)}}$$

where $\Phi(\cdot)$ is the CDF of the standard Normal.

Consider a planning horizon of $T \leq \infty$ periods and assume we discount future costs with a one-period discount factor $\beta \leq 1$.[4] Let

$$V_{(t)}(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)}) \;=\; \text{The expected minimal present value of costs incurred in periods } t, t+1, \cdots, T \text{ when starting in state } (\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)})$$

It is easily verified that the value functions satisfy the following exact DP:

$$\begin{aligned} &V_{(t)}(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)}) \\ &= \min_{\bar{\boldsymbol{x}}_{(t)}, \bar{X}_{(t)}, \bar{W}_{(t)} \geq 0} \Bigg\{ K_{(t)} \mathbb{I}_{\bar{W}_{(t)} > 0} + c_{(t)} \bar{W}_{(t)} + \sum_{j=1}^{J} \gamma_{j,(t)} (\bar{x}_{j,(t)} - x_{j,(t)}) \\ &\qquad + \sum_{j=1}^{J} Q_{j,(t)}(\bar{x}_{j,(t)}) + H_{(t)}(\bar{X}_{(t)}) \\ &\qquad + \beta \mathbb{E} V_{(t+1)} \left(\bar{\boldsymbol{x}}_{(t)} - \boldsymbol{u}_{(t)}, \bar{X}_{(t)}, \boldsymbol{W}_{(t+1)} \right) \Bigg\} \end{aligned} \tag{2}$$

$$\text{s.t.} \quad \bar{x}_{+,(t)} + \bar{X}_{(t)} = x_{+,(t)} + X_{(t)} + W_{(t-L)} \tag{3}$$

$$\bar{x}_{j,(t)} \leq \chi_{j,(t+\ell)}^{[\alpha_j]} \qquad j = 1, \cdots, J \tag{4}$$

$$\bar{X}_{(t)} \geq 0 \tag{5}$$

$$\bar{x}_{j,(t)} \geq x_{j,(t)} \qquad j = 1, \cdots, J \tag{6}$$

where $\boldsymbol{W}_{(t+1)} = [W_{(t-L+1)}, \cdots, W_{(t-1)}, \bar{W}_{(t)}]$, and $V_{(T-\ell+1)}(\cdot) = 0$.

In the minimand of (2), the first three terms represent the external order and internal shipping costs in period t; the next term denotes the expected discounted backlogging and holding costs at the retailers that are charged to period t, and the fifth term denotes the holding costs at the depot in period t.

Constraint (3) ensures that the total inventory in our system at the start of period t plus any order to be received there at the start of that period must be equal to the sum of all allocations made to the various facilities. Each of the shipment quantities must be non-negative (as implied by constraint (6)) as must the inventory allocated to the depot (constraint (5)). The remaining set of constraints (4) are identical to the inventory capacity constraints (1).

[4] When $T = \infty$ and $\beta = 1$, the value functions diverge. In this case, the long-run average cost value is the generally preferred criterion; see Section 7 for a treatment of this case.

5 A Lower Bound Approximation via Lagrangian Relaxation

In this section, we confine ourselves to finite horizon models, i.e. those with $T < \infty$. The required adaptations for infinite horizon models are covered in section 7. To simplify our exposition, we assume the retailers face identical shipment leadtimes, i.e., $\ell_j = \ell$ for all j. However, all results continue to hold when these leadtimes vary by retailer.

To obtain a tractable lower bound DP, we relax the set of constraints (6) using a Lagrangian relaxation, i.e., we implicitly allow for return shipments, back from the retailers to the depot. However, any negative allocation to retailer j is penalized at a rate $\lambda_{j,(t)}$, the Lagrangian multiplier associated with (6). This approach is similar to the relaxation approach in Federgruen and Zipkin (1984b), but using Lagrangian relaxation rather than a full relaxation, and applied to a DP that is more general in five essential ways, as discussed in the Introduction:

1. Our model allows for central inventories at the depot.
2. We add chance constraints to control the likelihood of an inventory overflow at any of the retailers, given arbitrary inventory capacity limits there. Similarly, we address an inventory capacity limit at the depot by an appropriate choice of the cost functions $H_{(t)}(\cdot)$.
3. We allow for arbitrary continuous demand distributions, rather than the Normal distributions Federgruen and Zipkin (1984b) confine themselves to; (see Footnote 1).
4. We allow for arbitrary cost parameters, rather than assuming that the retailers share the same holding and backlogging cost rates.
5. We allow for variable shipment costs from the depot to the retailers.

Our Lagrangian approach is also similar to that in Kunnumkal and Topaloglu (2008), in a model without storage capacity constraints and without fixed order costs. The Lagrangian relaxed problem is then shown to be equivalent to a sequence of nested one-dimensional DPs as opposed to a *single* one-dimensional DP. The inductive equivalency proof follows that in Clark and Scarf (1960).

Thus, to relax our dynamic program, we introduce a set of non-negative Lagrange multipliers $\{\lambda_{j,(t)}\}$ and construct a set of relaxed value functions $V^{\lambda}_{(t)}(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)})$ that satisfy the following recursions

$$
\begin{aligned}
& V^{\lambda}_{(t)}(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)}) \\
& \quad = \min_{\bar{\boldsymbol{x}}_{(t)}, \bar{X}_{(t)}, \bar{W}_{(t)}} \Big\{ K_{(t)} \mathbb{I}_{\bar{W}_{(t)}>0} + c_{(t)} \bar{W}_{(t)} \\
& \qquad + \sum_{j=1}^{J} (\lambda_{j,(t)} - \gamma_{j,(t)})(x_{j,(t)} - \bar{x}_{j,(t)}) \\
& \qquad + \sum_{j=1}^{J} Q_{j,(t)}(\bar{x}_{j,(t)}) + H_{(t)}(\bar{X}_{(t)}) \\
& \qquad + \beta \mathbb{E} V^{\lambda}_{(t+1)} \left(\bar{\boldsymbol{x}}_{(t)} - \boldsymbol{u}_{(t)}, \bar{X}_{(t)}, \boldsymbol{W}_{(t+1)} \right) \Big\} \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & \bar{x}_{+,(t)} + \bar{X}_{(t)} = x_{+,(t)} + X_{(t)} + W_{(t-L)} && (8) \\
& \bar{x}_{j,(t)} \le \chi^{[\alpha_j]}_{j,(t+\ell)} \qquad j = 1, \cdots, J && (9) \\
& \bar{X}_{(t)} \ge 0 && (10)
\end{aligned}
$$

Let $A_{(t)} \equiv x_{+,(t)} + X_{(t)}$ denote the total system-wide inventory level at the beginning of period t, which resides somewhere in the distribution system, either at the depot or the retailers, or in transit between them. Observe that, with the exception of the linear term $\sum_{j=1}^{J}(\lambda_{j,(t)} - \gamma_{j,(t)}) x_{j,(t)}$ in equation (7), $V^{\lambda}_{(t)}(\cdot)$ depends on the geographically disaggregated inventory information $(\boldsymbol{x}_{(t)}, X_{(t)})$ only via its aggregated value $A_{(t)}$.

Therefore, define new value functions

$$
\tilde{V}^{\lambda}_{(t)}(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)}) \equiv V^{\lambda}_{(t)}(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)}) - \sum_{j=1}^{J} (\lambda_{j,(t)} - \gamma_{j,(t)}) x_{j,(t)} \qquad (11)
$$

This means that

$$
V^{\lambda}_{(t+1)}(\boldsymbol{x}_{(t+1)}, X_{(t+1)}, \boldsymbol{W}_{(t+1)}) \equiv \tilde{V}^{\lambda}_{(t+1)}(\boldsymbol{x}_{(t+1)}, X_{(t+1)}, \boldsymbol{W}_{(t+1)}) + \sum_{j=1}^{J} (\lambda_{j,(t+1)} - \gamma_{j,(t+1)}) x_{j,(t+1)} \qquad (12)
$$

Now, substitute into (11) the right hand side of (7), and then substitute (12) into the resulting objective. We obtain the following recursions for the shifted value functions $\tilde{V}^{\lambda}$

$$\begin{aligned}
&\tilde{V}^{\lambda}_{(t)}(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)}) \\
&\quad = \min_{\bar{\boldsymbol{x}}_{(t)}, \bar{X}_{(t)}, \bar{W}_{(t)}} \Bigg\{ K_{(t)} \mathbb{I}_{\bar{W}_{(t)}>0} + c_{(t)} \bar{W}_{(t)} - \beta \sum_{j=1}^{J} (\lambda_{j,(t+1)} - \gamma_{j,(t+1)}) \mu_{j,(t)} \\
&\qquad + \sum_{j=1}^{J} \left[\beta(\lambda_{j,(t+1)} - \gamma_{j,(t+1)}) - (\lambda_{j,(t)} - \gamma_{j,(t)}) \right] \bar{x}_{j,(t)} \\
&\qquad + \sum_{j=1}^{J} Q_{j,(t)}(\bar{x}_{j,(t)}) + H_{(t)}(\bar{X}_{(t)}) \\
&\qquad + \beta \mathbb{E} \tilde{V}_{(t+1)} \left(\bar{\boldsymbol{x}}_{(t)} - \boldsymbol{u}_{(t)}, \bar{X}_{(t)}, \boldsymbol{W}_{(t+1)} \right) \Bigg\} \\
&\quad \text{s.t.} \quad \bar{x}_{+,(t)} + \bar{X}_{(t)} = x_{+,(t)} + X_{(t)} + W_{(t-L)} \\
&\qquad \bar{x}_{j,(t)} \le \chi^{[\alpha_j]}_{j,(t+\ell)} \qquad j = 1, \cdots, J \\
&\qquad \bar{X}_{(t)} \ge 0
\end{aligned}$$

Careful examination of the recursions above show that *these* value functions depend on the $J+L+1$-dimensional state vector $(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)})$ only through the aggregated $(L+1)$-dimensional vector $(A_{(t)}, \boldsymbol{W}_{(t)})$. Therefore, the *relaxed* DP can be formulated through the following value functions defined on $\mathbb{R}^{L+1}$. For $t = 1, \cdots, T-\ell$.

$$\begin{aligned}
&\hat{V}^{\lambda}_{(t)}(A_{(t)}, \boldsymbol{W}_{(t)}) \\
&\quad = \min_{\bar{\boldsymbol{x}}_{(t)}, \bar{X}_{(t)}, \bar{W}_{(t)}} \Bigg\{ \sum_{j=1}^{J} Q_{j,(t)}(\bar{x}_{j,(t)}) + H_{(t)}(\bar{X}_{(t)}) + K_{(t)} \mathbb{I}_{\bar{W}_{(t)}>0} + c_{(t)} \bar{W}_{(t)} \\
&\qquad + \sum_{j=1}^{J} \left[\beta(\lambda_{j,(t+1)} - \gamma_{j,(t+1)}) - (\lambda_{j,(t)} - \gamma_{j,(t)}) \right] \bar{x}_{j,(t)} \\
&\qquad + \beta \mathbb{E} \hat{V}_{(t+1)} \left(A_{(t)} - u_{+,(t)} + W_{(t-L)}, \boldsymbol{W}_{(t+1)} \right) \Bigg\} \\
&\qquad - \beta \sum_{j=1}^{J} (\lambda_{j,(t+1)} - \gamma_{j,(t+1)}) \mu_{j,(t)} \qquad (13) \\
&\quad \text{s.t.} \quad \bar{x}_{+,(t)} + \bar{X}_{(t)} = A_{(t)} + W_{(t-L)} \\
&\qquad \bar{x}_{j,(t)} \le \chi^{[\alpha_j]}_{j,(t+\ell)} \qquad j = 1, \cdots, J \\
&\qquad \bar{X}_{(t)} \ge 0 \qquad (14)
\end{aligned}$$

where $\boldsymbol{W}_{(t+1)} = [W_{(t-L+1)}, \cdots, W_{(t-1)}, \bar{W}_{(t)}]$ and $\hat{V}^{\lambda}_{(T-\ell+1)}(\cdot) = 0$.

Theorem 1. *For any* $\lambda = (\lambda_{j,(t)}) \in \mathbb{R}_+^{JT}$ *and any starting state* $(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)})$

$$\begin{aligned} \hat{V}^{\lambda}_{(t)}(x_{+,(t)} + X_{(t)}, \boldsymbol{W}_{(t)}) + \sum_{j=1}^{J} (\lambda_{j,(t)} - \gamma_{j,(t)}) x_{j,(t)} &= V^{\lambda}_{(t)}(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)}) \\ &\leq V_{(t)}(\boldsymbol{x}_{(t)}, X_{(t)}, \boldsymbol{W}_{(t)}) \end{aligned}$$

Proof. See Appendix **??** □

Moreover, the minimization in (13)–(14) over the action variables $\{\bar{\boldsymbol{x}}_{(t)}, \bar{X}_{(t)}, \bar{W}_{(t)}\}$ decomposes into two separate minimizations: one pertaining to the vector of inventory variables $[\bar{\boldsymbol{x}}_{(t)}, \bar{X}_{(t)}]$, and a second minimization over the order quantity $\bar{W}_{(t)}$. This observation allows us to simplify the recursion to:

$$\begin{aligned} \hat{V}^{\lambda}_{(t)}(A_{(t)}, W_{(t)}) = R^{\lambda}_{(t)}(A_{(t)} + \boldsymbol{W}_{(t-L)}) + \min_{\bar{W}_{(t)}} \Big\{ K_{(t)} \mathbb{I}_{\bar{W}_{(t)} > 0} + c_{(t)} \bar{W}_{(t)} \\ + \beta \mathbb{E} \hat{V}^{\lambda}_{(t+1)} \left(A_{(t)} - u_{+,(t)} + W_{(t-L)}, \boldsymbol{W}_{(t+1)} \right) \Big\} - \beta \sum_{j=1}^{J} (\lambda_{j,(t+1)} - \gamma_{j,(t+1)}) \mu_{j,(t)} \end{aligned}$$

where $R^{\lambda}_{(t)}(A)$ is the optimal objective value of the following problem

$$\min_{\bar{\boldsymbol{x}}_{(t)}, \bar{X}_{(t)}} \quad \sum_{j=1}^{J} Q_{j,(t)}(\bar{x}_{j,(t)}) + H_{(t)}(\bar{X}_{(t)}) + \left[\beta(\lambda_{j,(t+1)} - \gamma_{j,(t+1)}) - (\lambda_{j,(t)} - \gamma_{j,(t)}) \right] \bar{x}_{j,(t)} \tag{15}$$

$$\text{s.t.} \quad \bar{x}_{+,(t)} + \bar{X}_{(t)} = A \tag{16}$$

$$\bar{x}_{j,(t)} \leq \chi^{[\alpha_j]}_{j,(t+\ell)} \qquad j = 1, \cdots, J \tag{17}$$

$$\bar{X}_{(t)} \geq 0 \tag{18}$$

Since the functions $Q_{j,(t)}(\cdot)$ and $H_{(t)}(\cdot)$ are strictly convex, it is easily verified that $R^{\lambda}_{(t)}(\cdot)$ is strictly convex as well. We conclude that, in the relaxed DP, it is optimal to allocate the system-wide inventory $(A_{(t)} + W_{(t-L)})$ so as to minimize a linear function in the retailers' inventory allocations $\bar{\boldsymbol{x}}_{(t)}$, plus expected (discounted) costs in the very first period in which the allocations are received: for the depot, this means the current period, and for the retailers, it refers to the period one shipment leadtime after the current period. We refer to the convex program (15)-(18) as the *relaxed myopic allocation* problem. In the special settings addressed there, optimality of myopic allocations was derived for the relaxed DP in Federgruen and Zipkin (1984b).

The Lagrangian relaxation of the constraint set (6) thus allows us to replace a DP with a $(J+L+1)$-dimensional state space by one that is of dimension $(L+1)$. While a major improvement, this would still leave us with an intractable dynamic program, other than for the smallest values of the leadtime L. Moreover, the structure of an optimal policy would not be transparent. However, a standard substitution, identified by Scarf (1960), allows us to collapse the state space to a one-dimensional one, with a resulting DP that corresponds with a standard single-location inventory problem. Let

$$\begin{aligned} I_{(t)} &= \text{The system-wide inventory } \textit{position} \text{ at the start of period } t \\ &= A_{(t)} + \sum_{\tau=t-L}^{t-1} W_{(\tau)} \\ &= \text{all inventory currently in the distribution system } (A_{(t)}) \text{ plus} \\ &\quad\ \text{all orders with the outside supplier that are still outstanding} \end{aligned}$$

Similar to the accounting scheme used in the construction of the $Q_{j,(t)}(\cdot)$ functions, instead of assigning at the beginning of period t the cost $R^{\lambda}_{(t)}(A_{(t)} + W_{(t-L)})$ as a function of the system-wide inventory known at that time, assign to period t the discounted expectation of $R^{\lambda}_{(t+L)}(A_{(t+L)}+W_{(t)})$, the cost value an order lead time later. Thus, noting that $A_{(t+L)} + W_{(t)} = I_{(t)} + \bar{W}_{(t)} - \hat{u}_{+,(t)}$, we charge to period t an inventory cost

$$G^{\lambda}_{(t)}(I_{(t)} + \bar{W}_{(t)}) \equiv \beta^L \mathbb{E} R^{\lambda}_{(t+L)}(I_{(t)} + \bar{W}_{(t)} - \hat{u}_{+,(t)})$$

Once again, $G^{\lambda}_{(t)}(\cdot)$ is a strictly convex function, since $R^{\lambda}_{(t+L)}(\cdot)$ is. With this accounting scheme, all inventory costs charged to period t depend only on the system-wide inventory position $I_{(t)}$ and the order placed in that period.

This forward accounting scheme covers the expected discounted value of the holding and backorder costs pertaining to periods $L+1, L+2, \cdots$. Those pertaining to the initial periods $t = 1, \cdots, L$ are, however, omitted from this formulation. Indeed, these inventory costs cannot be controlled, since any orders from from period 1 onward impact the system only in period $(L+1)$ and beyond. These early costs are therefore independent of our decisions, and can be given as a single function of the initial state $(A_{(1)}, \boldsymbol{W}_{(1)})$:

$$\nu^{\lambda}(A_{(1)}, \boldsymbol{W}_{(1)}) = \sum_{t=1}^{L} \beta^{t-1} \mathbb{E} R^{\lambda}_{(t)} \left(A_{(1)} + \sum_{\tau=1-L}^{t-L} W_{(\tau)} - \sum_{\tau=1}^{t} u_{+,(\tau)} \right)$$

Thus,

$$\hat{V}^{\lambda}_{(1)}(A_{(1)}, \boldsymbol{W}_{(1)}) = \nu^{\lambda}(A_{(1)}, \boldsymbol{W}_{(1)}) + \sum_{\tau=1}^{T} \sum_{j=1}^{J} \beta^{\tau} (\lambda_{j,(\tau)} - \gamma_{j,(\tau)}) \mu_{j,(\tau)} + \dot{V}^{\lambda}_{(1)} \left(A_{(1)} + \sum_{\tau=1-L}^{0} W_{(\tau)} \right)$$

where

$$\dot{V}^{\lambda}_{(t)}(I_{(t)}) = \min_{\bar{W}_{(t)}} \left\{ G^{\lambda}_{(t)}(I_{(t)} + \bar{W}_{(t)}) + K_{(t)}\mathbb{I}_{\bar{W}_{(t)}\geq 0} + c_{(t)}\bar{W}_{(t)} + \beta\mathbb{E}\dot{V}^{\lambda}_{(t+1)}(I_{(t)} + \bar{W}_{(t)} - u_{+,(t)}) \right\} \quad (19)$$

with $\dot{V}^{\lambda}_{(T-\ell-L+1)}(\cdot) = 0$. Combining (19) with the lower bound in Theorem 1, we get

$$V_{(1)}(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)}) \geq V^{\lambda}_{(1)}(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)}) = \sum_{j=1}^{J}(\lambda_{j,(1)} - \gamma_{j,(1)})x_{j,(1)}$$
$$+ \sum_{\tau=1}^{T}\sum_{j=1}^{J}\beta^t(\lambda_{j,(\tau)} - \gamma_{j,(\tau)})\mu_{j,(\tau)} + \nu^{\lambda}(x_{+,(1)} + X_{(1)}, \boldsymbol{W}_{(1)}) + \dot{V}^{\lambda}\left(x_{+,(1)} + X_{(1)} + \sum_{\tau=1-L}^{0} W_{(\tau)}\right) \quad (20)$$

The first two terms in the lower bound (20) are available in closed form; the third term requires the solution of the relaxed myopic distribution problems, which can be done very efficiently; see section 6 for a description of effective algorithms. Finally, evaluation of the last term reduces to the solution of a DP with a *one-dimensional* state space only. Moreover, this DP may be interpreted as one pertaining to a *single* location inventory system, which implies a (time-dependent) (s, S) policy is optimal.

We conclude

Theorem 2. *Assume $K_{(t)} \geq \beta K_{(t+1)}$ for all $t = 1, \cdots, T$.*

(a) *In the relaxed DP, it is optimal to place orders from the supplier in accordance with an $(s_{(t)}, S_{(t)})$ policy acting on the system-wide inventory position $I_{(t)}$.*

(b) *Fix $t = 1, \cdots, T - (L + \ell + 1)$. In the relaxed DP, it is optimal to allocate the system-wide inventory $(A_{(t)} + W_{(t-L)})$ among the retailers and the depot in accordance with the relaxed myopic allocation problem (15)-(18).*

Proof. (a) Follows immediately from Scarf (1960); see also standard textbooks such as Zipkin (2000). (b) See the derivation in the text. Note that the parameter restriction $K_{(t)} \geq \beta K_{(t+1)}$ is required even in a single echelon, single location model to guarantee the K-convexity of the value functions. This property is essential to obtain the optimality of (s, S)-policies in finite-horizon models. □

To solve the Lagrangian dual, i.e. to find the *largest* lower bound

$$\max_{\lambda\geq 0} V^{\lambda}_{(1)}(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)})$$

we need to embed (20) in an unconstrained maximization over all non-negative λ vectors.

Theorem 3, below, shows that the lower bound in (20) is a concave function of λ. This implies that any standard steepest ascent method is guaranteed to converge to the optimal multiplier vector $\lambda^\star$, see, for example, Bertsekas (1999), section 6.3, or Boyd and Vandenberghe (2004), Chapter 9. Such methods move, in each iteration, from the current solution in the direction of a supergradient there. This means that at each iteration, the computational requirement consists of the evaluation of (i) the function value $V^\lambda_{(1)}(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)})$ and (ii) a supergradient with respect to λ. We have shown that the *former* essentially reduces to the evaluation of a DP with a one-dimensional state space. Theorem 3 identifies a simple formula by which the supergradients at any point $\lambda = \lambda^0$ can be computed, once the optimal replenishment and allocation policy in the relaxed DP, under $\lambda = \lambda^0$, have been identified.

We will need the following notation:

- Let $\bar{\boldsymbol{x}}^{\star,\lambda}_{(t)}(\bar{A})$ represent the unique optimal vector of decision variables $\bar{\boldsymbol{x}}_{(t)}$ in optimization problem (15)–(18) for $R^\lambda_{(t)}(\bar{A})$.
- Let $\bar{W}^{\star,\lambda}_{(t)}(I_{(t)})$ denote the optimal value of the variable $\bar{\boldsymbol{W}}_{(t)}$ in the optimization problem (19) for $\dot{V}^\lambda_{(t)}(I_{(t)})$. i.e.,

$$\bar{W}^{\star,\lambda}_{(t)}(I_{(t)}) = \begin{cases} S_{(t)} - I_{(t)} & \text{if } I_{(t)} \le s_{(t)} \\ 0 & \text{otherwise} \end{cases}$$

  where $(s_{(t)}, S_{(t)})$ are the parameters of the (s, S) policy to be used optimally in period t; see Theorem 2.
- Define a stochastic process $\left\{\iota^\lambda_{(t)} : t = 1, \cdots, T - L - \ell\right\}$ by fixing the value of $\iota^\lambda_{(1)}$, and letting

$$\iota^\lambda_{(t)} = \iota^\lambda_{(t-1)} + \bar{W}^{\star,\lambda}_{(t-1)}(\iota^\lambda_{(t-1)}) - u_{+,(t-1)}$$

  The stochastic process $\{\iota^\lambda_{(t)}\}$ represents the total system-wide inventory position process when the total system-wide inventory at the start of period 1 is $\iota^\lambda_{(1)}$ and decisions are made according to the optimal policy in the relaxed DP with Lagrange multipliers λ.

We will need the following Lemma, the proof of which is given in Appendix **??**.

Lemma 1. *For any given value of $\bar{A}$, I, and $(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)})$,*

(a)

$$\frac{\partial R^{\lambda}_{(t)}(\bar{A})}{\partial \lambda_{j,(\tau)}} = \begin{cases} -\bar{x}^{\star,\lambda}_{j,(t)}(\bar{A}) & \tau = t \\ \beta \bar{x}^{\star,\lambda}_{j,(t)}(\bar{A}) & \tau = t+1 \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$\frac{\partial G^{\lambda}_{(t)}(I)}{\partial \lambda_{j,(\tau)}} = \begin{cases} -\beta^{L}\mathbb{E}\bar{x}^{\star,\lambda}_{j,(t+L)}(I - \hat{u}_{+,(t)}) & \tau = t+L \\ \beta^{L+1}\mathbb{E}\bar{x}^{\star,\lambda}_{j,(t+L)}(I - \hat{u}_{+,(t)}) & \tau = t+L+1 \\ 0 & \text{otherwise} \end{cases}$$

(c)

$$\frac{\partial \nu^{\lambda}(A_{(1)}, \boldsymbol{W}_{(1)})}{\partial \lambda_{j,(\tau)}} = \begin{cases} -\mathbb{E}\bar{x}^{\star,\lambda}_{j,(1)}(A_{(1)} + W_{(1-L)} - u_{+,(1)}) & \tau = 1 \\ +\beta^{\tau-1}\mathbb{E}\bar{x}^{\star,\lambda}_{j,(\tau-1)}(A_{(1)} + \sum_{\tau'=1-L}^{\tau-1-L} W_{(\tau')} - \sum_{\tau'=1}^{\tau-1} u_{+,(\tau')}) & \\ \quad -\beta^{\tau-1}\mathbb{E}\bar{x}^{\star,\lambda}_{j,(\tau)}(A_{(1)} + \sum_{\tau'=1-L}^{\tau-L} W_{(\tau')} - \sum_{\tau'=1}^{\tau} u_{+,(\tau')}) & \tau = 2, \cdots, L \\ \beta^{L}\mathbb{E}\bar{x}^{\star,\lambda}_{j,(L)}(A_{(1)} + \sum_{\tau'=1-L}^{0} W_{(\tau')} - \sum_{\tau'=1}^{L-1} u_{+,(\tau')}) & \tau = L+1 \\ 0 & \text{otherwise} \end{cases}$$

These simple expressions enable an efficient evaluation of a supergradient.

Theorem 3. *For any given value of* $(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)})$,

(a) *The lower bound* $V^{\lambda}_{(1)}(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)})$ *is a concave function of* λ

(b) $\nabla_{\lambda} V^{\lambda}_{(1)}(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)})$ *is a supergradient of* $V^{\lambda}_{(1)}(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)})$ *with respect to* λ, *where*

$$\begin{aligned}\left[\nabla_{\lambda} V^{\lambda}_{(1)}(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)})\right]_{j,(t)} &= x_{j,(1)}\mathbb{I}_{t=1} + \beta^{L}\mu_{j,(t)} + \frac{\partial \nu^{\lambda}(x_{+,(1)} + X_{(1)}, \boldsymbol{W}_{(1)})}{\partial \lambda_{j,(t)}} \\ &+ \mathbb{E}\left[\frac{\partial G^{\lambda}_{(t-L)}(\iota^{\lambda}_{(t-L)} + \bar{W}^{\star,\lambda}_{(t-L)}(\iota^{\lambda}_{(t-L)}))}{\partial \lambda_{j,(t)}}\right. \\ &\left. + \frac{\partial G^{\lambda}_{(t-L-1)}(\iota^{\lambda}_{(t-L-1)} + \bar{W}^{\star,\lambda}_{(t-L-1)}(\iota^{\lambda}_{(t-L-1)}))}{\partial \lambda_{j,(t)}} \;\middle|\; \iota^{\lambda}_{(1)} = x_{+,(1)} + X_{(1)} + \sum_{\tau=1-L}^{0} W_{(\tau)}\right]\end{aligned}$$

Proof. See Appendix **??**. □

5.1 Computational Complexity of the Lower Bound for a Given Vector of Lagrange Multipliers

As mentioned, at each iteration of the steepest ascent method to solve the Lagrangian dual, the computational requirement consists of the evaluation of (i) the function value $V^{\lambda}_{(1)}(\boldsymbol{x}_{(1)}, X_{(1)}, \boldsymbol{W}_{(1)})$ and (ii) a supergradient with respect to λ, both for the current vector of Lagrange multipliers λ. Theorem 3 shows that, once the bound value itself has been evaluated, the supergradient can be computed with minimal additional effort. Moreover, evaluations of the bound itself essentially reduces to the solution of a single dynamic program with a state and action space that are both of single dimension. This is in contrast to the approach in Kunnumkal and Topaloglu (2008) which, for any given vector λ, requires the solution of a nested sequence of $(J+1)$ such dynamic programs, one for each retailer, and a final dynamic program that invokes the value functions of the various retailer problems. All of these dynamic programs have a similar size state and action space.

To achieve maximum efficiency, it is advisable for the function values $\{R_t(A_t) : A_t \geq 0\}$ be computed upfront. $R_t(A)$ represents the value of a non-linear allocation problem of specific structure. More specifically, the allocation problem minimizes a separable convex objective function, subject to a single budget constraint and simple upper bounds – but no lower bounds – on the individual allocation variables. Several very efficient solution methods exist for this class of problems, see e.g. Zipkin (1980). Section 5 there describes how the full range $\{R_t(A_t) : A_t \geq 0\}$ can be obtained efficiently with a computational effort comparable to that required to solve a *single* value $R_t(A^0)$ for some specific A^0. We review this method at the end of Section 6 (see problem (P) there). The computational effort required by the solution method depends primarily on the effort to compute derivatives and inverses of derivatives of each of the non-linear term functions in the objective. In our case, these are relatively simple holding and backlogging cost functions. Kunnumkal and Topaloglu's method requires the repeated solution of a similar allocation problem in its depot-based dynamic program; however, in that allocation problem, the terms in the objective function depend on the value functions of underlying retailer dynamic programs, the derivatives and inverses of derivatives of which are harder to compute.

6 An Upper Bound: a Heuristic Policy

In this section, we propose a specific feasible strategy of simple structure to govern the distribution system. The expected cost, under this policy, is of course an *upper bound* on the optimal cost value, and this for any starting state and time horizon. While relatively simple, its cost performance is still too difficult to be assessed analytically. Instead, we evaluate this with Monte Carlo simulations. Extensive numerical studies reported in section 8 compare the *lower bound*, resulting from the approximate DP in the previous section, with this *upper* bound.

A replenishment strategy consists of two components. (a) An *order policy* which dynamically prescribes when a system-wide order is to be placed with the external supplier, and of what size; (b) an *allocation policy* which, in each period, prescribes how the available inventory at the depot, including any newly received order with the supplier, is to be allocated among the depot and each of the retailers. As to the order policy, it is natural to adopt the $\{(s_t, S_t) : t = 1, \cdots, T\}$ policy which is optimal in the approximate DP, see Theorem 2 (or Theorem 4, below, if an infinite horizon stationary model is to be solved).

In the infinite horizon model with stationary parameters and demand distributions, efficient algorithms have been proposed to compute the optimal policy parameters $(s^\star, S^\star)$ and the associated expected cost value, see e.g. Zheng and Federgruen (1991). In finite horizon settings, where a different policy parameter pair (s_t, S_t) is to be determined for each period, a regular one-dimensional DP needs to be solved, albeit with various possible simplifications. For example, if for a given inventory position value I^0, it is optimal not to place an order, the same is true for any $I \geq I^0$. If, on the other hand, it is optimal to place an order which elevates the inventory position to some level S, then the same is true for all $I \leq I^0$.

As to the *allocation* policy, the myopic policy (15)-(18) which defines the functions $R_{(t)}(\tilde{A})$ and is optimal in the relaxed DP (see Theorem 2) could be employed as well. To make this into a feasible policy, we reinstate the relaxed constraints (6). Assuming the amount of inventory available for allocation is given by A, this gives rise to the following optimization problem

(MA) - *Myopic Allocation*

$$\min_{\bar{\boldsymbol{x}}_{(t)}, \bar{X}_{(t)}} \quad \left\{ \sum_{j=1}^{J} Q_{j,(t)}(\bar{x}_{j,(t)}) + H_{(t)}(\bar{X}_{(t)}) + \gamma_{j,(t)}(\bar{x}_{j,(t)} - x_{j,(t)}) \right\} \tag{21}$$

$$\text{s.t.} \quad \bar{x}_{+,(t)} + \bar{X}_{(t)} = A \tag{22}$$

$$\bar{x}_{j,(t)} \leq \chi^{[\alpha_j]}_{j,(t+\ell)} \qquad j = 1, \cdots, J \tag{23}$$

$$\bar{x}_{j,(t)} \geq x_{j,(t)} \qquad j = 1, \cdots, J \tag{24}$$

$$\bar{X}_{(t)} \geq 0 \tag{25}$$

However, myopic allocations, intrinsically, focus on the very *first* period in which retailer allocations arrive at their destinations. In particular, for sample paths on which the total system-wide inventory level $(A_{(t)}+W_{(t-L)})$ is relatively low, this myopic allocation policy may pull almost all the inventory towards the retailers through irreversible allocations without regard for the coverage needs and consequences in subsequent periods. This is particularly problematic when the next opportunity to replenish the retailers' inventory positions is several periods away, because nothing or relatively little is left at the depot until the next order from the external supplier arrives there. We therefore consider a generalization of the Myopic Allocation problem (MA) where the allocations are made to optimize expected costs over a time window of $\kappa > 1$ periods, assuming no other allocations are made during this time window:

(κA) - *κ-Period Allocation*

$$\min_{\bar{\boldsymbol{x}}_{(t)},\bar{X}_{(t)}} \quad \left\{ \sum_{j=1}^{J} Q^{(\kappa)}_{j,(t)}(\bar{x}_{j,(t)}) + H^{(\kappa)}_{(t)}(\bar{X}_{(t)}) + \gamma_{j,(t)}(\bar{x}_{j,(t)} - x_{j,(t)}) \right\}$$

$$\text{s.t.} \qquad (22)\text{–}(25)$$

where

$$Q^{(\kappa)}_{j,(t)}(\bar{x}_{j,(t)}) = \mathbb{E} \sum_{\tau=t+\ell_j}^{t+\ell_j+\kappa-1} \beta^{\tau-t} \left\{ h_{j,(\tau)} \left(\bar{x}_{j,(t)} - \sum_{\tau'=t}^{t+\tau} u_{j,(\tau')} \right)^{+} p_{j,(\tau)} \left(\bar{x}_{j,(t)} - \sum_{\tau'=t}^{t+\tau} u_{j,(\tau')} \right)^{-} \right\}$$

$$H^{\kappa}_{(t)}(\bar{X}_{(t)}) = \sum_{\tau=t}^{t+\kappa-1} H_{(\tau)}(\bar{X}_{(t)})$$

It is, again, easy to verify that the functions $Q^{(\kappa)}_{j,(t)}(\cdot)$ and $H^{(\kappa)}_{(t)}(\cdot)$ are strictly convex.

There are several possible choices for the length of the time window κ. In describing them, we confine ourselves to settings where a *stationary* (s, S) policy is used, see e.g. Theorem 4. In such a setting, the system inventory position process $\{I_{(t)}\}$ is regenerative. Let

$$\begin{aligned} M(x) \quad &= \quad \text{The expected number of periods until the next order is placed} \\ &\qquad \text{when the current initial system-wide inventory position is } (s+x) \\ &= \quad 1 + \int_0^x M(x-u)\, \mathrm{d}F_+(u) \end{aligned} \tag{26}$$

where $F_+(\cdot)$ denotes the CDF of the system-wide demand u_+. In other words, $M(\cdot)$ denotes the renewal function associated with the random variable u_+. The expected cycle length, i.e. the expected number of periods between orders is then given by $M(S-s)$.

One possible choice for κ, therefore, is $\kappa_1 \equiv \max\{1, M(S-s) - (t-\Lambda)\}$, where

$$\Lambda \equiv \max\left\{\tau \le t : W_{(\tau-L)} > 0\right\}$$

denotes the last period prior to the current period in which an order arrived.

κ_1 is a stationary estimate of the remaining time until the next order is to arrive at the depot. This estimate ignores information about the current pipeline of outstanding orders

$$[W_{(t-L+1)}, W_{(t-L+2)}, \cdots, W_{(t)}]$$

as well as the actual system-wide inventory position $I_{(t)}$. The first *as of yet unscheduled* order is expected to arrive in period $(t + M(I_{(t)} - s) + L)$. However, that order is of course preceded by any order already in the pipeline, resulting in the specification, which we use in the remainder of this paper and in our numerical studies

$$\kappa_2 \equiv \max\left\{1, \min\{\{n : W_{(t+n-L)} > 0\}, \lfloor M(I_{(t)} - s)\rfloor + L\}\right\}$$

with the convention that $+\infty$ is the minimum value of an empty set.

The κ-period allocation strategy (κA), almost invariably, results in significant cost savings compared to the standard Myopic Allocation (MA) strategy. At the end of Section 8, we provide a detailed illustration of the performance of both allocation strategies, explaining why major cost savings can be expected.

Federgruen and Zipkin (1984a) have proposed allocation policies of the (κ)-type, in the context of the model considered there. However, the authors consider its solution too complex to be a realistic alternative, and suggested focusing on a *specific* period within the time window of κ periods, as opposed to optimizing the aggregate expected costs.

Both the Myopic Allocation problem (MA) and the allocation problem defined by the functions $R_t(A)$ in the lower bound approach in section 5 are special cases of the allocation problem (κA), at least from a structural point of view.

The allocation problem (κA) consists of minimizing a separable strictly convex function subject to a budget constraint and upper and lower bounds for the individual decision variables:

$$\begin{aligned}
\text{(P)}\ z^\star \quad &= \min \quad \sum_{n=1}^{N} q_n(x_n) \\
&\text{s.t.} \quad \sum_{n=1}^{N} x_n = b \qquad (27) \\
&\qquad\quad L_n \le x_n \le U_n \qquad n = 1, \cdots, N \qquad (28)
\end{aligned}$$

where each $q_n(\cdot)$ is a strictly convex function for all $n = 1, \cdots, N$. Without loss of generality, we assume that $\sum_{n=1}^{N} L_n \le b \le \sum_{n=1}^{N} U_n$.

Efficient algorithms for this class of problems were proposed by Bitran and Hax (1981), Bodin (1969), Luss and Gupta (1974); see also Ohuchi and Kaji (1980) and Zipkin (1980). We have applied the following basic algorithm: since (P) is a strictly convex program, it is well known that its unique optimal solution $\boldsymbol{x}^\star$ can be obtained by finding the corresponding optimal solution $z^\star(\pi)$ to the following Lagrangian relaxation, with $-\infty < \pi < \infty$.

$$(\mathrm{P}_\pi)\ z^\star(\pi) = \min \sum_{n=1}^{N} q_n(x_n) + \pi \left[b - \sum_{n=1}^{N} x_n \right] \text{ s.t. } (28)$$

and computing the value of π for which $\boldsymbol{x}^\star(\pi)$ satisfies the relaxed constraint (27), $\sum_{n=1}^N x_n^\star(\pi) = b$. (Clearly, $z^\star \geq z^\star(\pi)$ for all π, since the feasible region of (P_π) includes that of (P); let $\pi^\star$ be such that $\sum_{n=1}^N x_n^\star(\pi^\star) = b$ (i.e., $\boldsymbol{x}^\star(\pi^\star)$ is a feasible solution for (P)). Then, $z^\star \geq z^\star(\pi^\star) = \sum_{n=1}^N q_n(x_n^\star(\pi^\star)) \geq z^\star$, which proves the optimality of $\boldsymbol{x}^\star(\pi^\star)$.)

Let $q_n'(\cdot)$ denote the strictly increasing derivative of the function $q_n(\cdot)$ and $q_n'^{-1}(\cdot)$ its inverse. The relaxed problem (P_π) decomposes into N separate optimizations of single variable functions. More specifically, $\boldsymbol{x}_n^\star(\pi)$ is the unique minimum of the function $(q_n(x_n) - \pi x_n)$ over $x_n \in [L_n, U_n]$. Thus, in view of the strict convexity of $q_n(\cdot)$,

$$x_n^\star(\pi) = \min \left\{ U_n, \max[L_n, q_n'^{-1}(\pi)] \right\} \tag{29}$$

Thus, the original problem (P) can be solved by identifying the unique value $\pi^\star$ for which $\sum_{n=1}^N x_n^\star(\pi) - b$, i.e. solving the following equation in π

$$\sum_{n=1}^{N} \min \left\{ U_n, \max[L_n, q_n^{-1}(\pi)] \right\} = b \tag{30}$$

The search for a root of (30) may be confined to the interval $[\min_n q_n'(L_n), \max_n q_n'(U_n)]$ since for $\pi < \min_n q_n'(L_n)$, we have $q_n'^{-1}(\pi) < L_n$ for all $n = 1, \cdot, N$, and so the LHS of (30) will be given by $\sum_{n=1}^N L_n < b$. Similarly, when $\pi > \max_n q_n'(U_n)$, the LHS of (30) will be given by $\sum_{n=1}^N U_n > b$. Moreover, over the given interval, at least one of the terms on the LHS of (30) is strictly increasing. This implies the existence of a *unique* root over this interval which can be found using a standard bisection method.

Often, problem (P) needs to be solved, parametrically, with the budget size b varying systematically over the interval $[\sum_n L_n, \sum_n U_n]$. This applies, for example, when the full range of values $\{R_t(A)\}$ need to be computed as part of the lower bound computation in section 5.

Since $q_n'^{-1}(\cdot)$ is strictly increasing for all n, it follows that $\pi^\star(b)$, implicitly defined by (30), is a strictly increasing function on $[\sum_n L_n, \sum_n U_n]$, hence with a strictly increasing inverse function

$b = \pi^{\star -1}(\pi)$. The parametric optimization of (P) is thus most easily obtained by varying π on the interval $[\min_n q_n'(L_n), \max_n q_n'(U_n)]$. For any given value of π, the optimal associated allocation $\{x_n^\star(\pi), n = 1, \cdots, N\}$ is given by (29) and $b(\pi) = \sum_n x_n^\star(\pi)$. The evaluation of (29) can be further simplified by pre-computing and sorting the values $\{q_n'(L_n), q_n'(U_n), n = 1, \cdots, N\}$ in $O(N \log N)$ time. As π is increased on any of the intervals between a consecutive pair of breakpoints, the sets $I_L^\star = \{n : x_n^\star \leq L_n\}$, $I_m^\star = \{n : L_n \leq x_n^\star \leq U_n\}$ and $I_I^\star = \{n : x_n^\star \leq U_n\}$ remain invariant, thus avoiding the double comparison in (29). Moreover, as π is moved from one interval to the next, exactly *one* of the variables moves from $I_L^\star$ to $I_m^\star$ or from $I_m^\star$ to $I_U^\star$.

Finally, when inventory levels are discrete and integer valued, problem (P) may also be solved to optimality by a simple greedy procedure; starting with the solution $\boldsymbol{x}^0 = [L_1, \cdots, L_N]$, we allocate each of the remaining $(b - \sum_{n=1}^N L_n)$ units sequentially to any variable x_n that is not yet at its upper bound U_n and for which the marginal cost changed is minimized. Since the feasible region of (P) is the base of a polymatroid , it is well known that this greedy procedure generates an optimal solution, see Federgruen and Groenevelt (1986).

7 Infinite Horizon Models

In this section, we consider a stationary model, in which the cost and capacity parameters, as well as all demand distributions, are time-independent. We therefore omit the time index in our notation. Based on standard results, see, for example, Bertsekas and Shreve (1996) it is possible to show that as the planning horizon $T \to \infty$, the value function $V_{(1)}(\boldsymbol{x}, X, \boldsymbol{W})$ converges to an infinite horizon value function $V_\infty(\boldsymbol{x}, X, \boldsymbol{W})$ and that this function is a solution to the optimality equation

$$\begin{aligned} V_\infty(\boldsymbol{x}, X, \boldsymbol{W}) \quad &= \min_{\bar{\boldsymbol{x}}, \bar{X}, \bar{W} \geq 0} \quad \Bigg\{ K\mathbb{I}_{\bar{W}>0} + c\bar{W} + \sum_{j=1}^J \gamma_j(\bar{x}_j - x_j) + \sum_{j=1}^J Q_j(\bar{x}_j) + H(\bar{X}) \\ &\qquad\qquad + \beta\mathbb{E}V_\infty\left(\bar{\boldsymbol{x}} - \boldsymbol{u}, \bar{X}, \boldsymbol{W}_{(+1)}\right) \Bigg\} \\ &\text{s.t.} \qquad \bar{x}_+ + \bar{X} = x_+ + X + W_{(-L)} \\ &\qquad\qquad \bar{x}_j \leq \chi_j^{[\alpha_j]} \qquad j = 1, \cdots, J \\ &\qquad\qquad \bar{X} \geq 0 \\ &\qquad\qquad \bar{x}_j \geq x_{j,(t)} \qquad j = 1, \cdots, J \end{aligned} \tag{31}$$

where $W_{(-\tau)}$ denotes the order placed τ periods ago and $\boldsymbol{W}_{(+1)} = [W_{(-L+1)}, \cdots, W_{(-1)}, \bar{W}]$.

As before, we relax the constraints (31) which ensure that the allocations to all retailers are non-negative, via Lagrangian relaxation, associating a non-negative Lagrange multiplier λ_j to the constraint for retailer j. Proceeding as in the finite horizon case, this results in a *lower bound* function $V_\infty^\lambda(\boldsymbol{x}, X, \boldsymbol{W})$ which can be expressed in terms of the solution of an infinite horizon DP, indeed one with a $(L+1)$-dimensional state space. More specifically, analogous to Theorem 1, we get

$$V_\infty(\boldsymbol{x}, X, \boldsymbol{W}) \geq V_\infty^\lambda(\boldsymbol{x}, X, \boldsymbol{W}) = \sum_{j=1}^{J}(\lambda_j - \gamma_j)x_j + \frac{\beta}{1-\beta}\sum_{j=1}^{J}(\lambda_j - \gamma_j)\mu_j + \hat{V}_\infty^\lambda(x_+ + X, \boldsymbol{W})$$

where $\hat{V}_\infty^\lambda(A, \boldsymbol{W})$ satisfies the infinite-horizon optimality equation

$$\hat{V}_\infty^\lambda(A, \boldsymbol{W}) = \min_{\bar{\boldsymbol{x}}, \bar{X}, \bar{\boldsymbol{W}} \geq 0} \left\{ \sum_{j=1}^{J} Q_j(\bar{x}_j) + H(\bar{X}) + K\mathbb{I}_{\bar{W}>0} + c\bar{W} + \right.$$
$$\left. (1-\beta)\sum_{j=1}^{J}(\lambda_j - \gamma_j)\bar{x}_j + \beta\mathbb{E}\hat{V}_\infty^\lambda(A - u_+ + W_{(-L)}, \boldsymbol{W}_{(+1)} \right\}$$

Furthermore, $\hat{V}_\infty^\lambda$ can again be simplified as follows:

$$\hat{V}_\infty^\lambda(A, \boldsymbol{W}) = \nu^\lambda(A, \boldsymbol{W}) + \dot{V}_\infty^\lambda\left(A + \sum_{\tau=-L}^{1-L} \boldsymbol{W}_{(\tau)}\right)$$

where $\dot{V}_\infty^\lambda(\cdot)$ satisfies the optimality condition of the *single dimensional* DP

$$\dot{V}_\infty^\lambda(I) = \min_{\bar{W} \geq 0} \left\{ K\mathbb{I}_{\bar{W} \geq 0} + c\bar{W} + G(I + \bar{W}) + \beta\mathbb{E}\dot{V}^\lambda(I + \bar{W} - u_+) \right\} \tag{32}$$

Theorem 4 below show that a *stationary* (s, S) policy acting on the total inventory position is optimal in this DP. The Lagrangian relaxation in the *infinite* horizon model only depends on J Lagrange multipliers as opposed to JT Lagrange multipliers in the finite horizon case. This significantly simplifies the task of solving the Lagrangian dual i.e. of minimizing the function $V_\infty^\lambda(\boldsymbol{x}, X, \boldsymbol{W})$ over $\lambda \in \mathbb{R}_+^J$.

Finally, consider the same stationary infinite horizon model, except that the long-run average cost is to be minimized. In this model, it is well known that the long-run average shipment costs (from the depot to the retailers) amount to $\sum_{j=1}^{J}\gamma_j\mu_j$ under any stable policy (any stable policy must, on average, ship μ_j units per period to retailer j, or incur ever larger inventories or backlogs). This implies that, without loss of generality, variable shipment costs can be ignored in this version of the model. Similarly, assume that, as a final step to simplifying the Markov Decision Process with a $(J + L + 2)$-dimensional space, we relax the constraints $\bar{x}_j \geq x_j$, and instead add a penalty $\sum_{j=1}^{J} \lambda_j(\boldsymbol{x}_j - \bar{x}_J)$ to the one step expected cost functions, for a given vector $\lambda \in \mathbb{R}_+^J$ of Lagrange multipliers. Let $g^\star(\lambda)$ denote the long-run average cost value of the Lagrangian relaxed MDP. One can easily verify that

$$g^\star(\lambda) = g^\star(0) - \sum_{j=1}^{J} \lambda_j \mu_j$$

(By the argument above, the long-run average value of the Lagrangian penalty equals $-\sum_{j=1}^{J} \lambda_j \mu_j$ under any stable policy, since the long-run average shipment quantity to retailer j is equal to μ_j). It therefore follows that in the Lagrangian dual

$$\max_{\lambda \geq 0} g^\star(\lambda) = g^\star(0)$$

In other words, under the long-run average cost criterion, Lagrangian relaxation results in the *same* bound as simple relaxation. Following the relaxation steps in Section 5, the lower bound $g^\star(0) = \max_{\lambda \geq 0}$ can be computed by solving an MDP on a one-dimensional state space.

The model is characterized by a relative value function $\hat{V}(\cdot)$ and a constant $g^\star$, representing the optimal long-run average cost value, which satisfy the optimality equation

$$\hat{V}(I) = \min_{\bar{W} \geq 0} \left\{ K\mathbb{I}_{\bar{W}>0} + c\bar{W} + G(I) - g^\star + \mathbb{E}\hat{V}(I + \bar{W} - u_+) \right\} \tag{33}$$

The following Theorem follows immediately from classical results, see Iglehart (1963), Veinott (1966), or Zipkin (2000).

Theorem 4. *Consider the stationary infinite horizon model.*

(a) *Assume we want to minimize the present value of infinite horizon costs with a discounting function $\beta < 1$. Then an (s, S) policy achieves the minima in (32) and is optimal in the relaxed DP.*

(b) *Assume we want to minimize the long-run average costs. An (s, S) policy achieves the minima in (33) and is optimal in the relaxed DP.*

8 Numerical Study

In this section, we describe an extensive numerical study to assess the accuracy of the lower bound DP derived in section 5 and the optimality gap of the proposed heuristic policy, described in section 6. The first study reports on 7,776 problem instances, and the second on an additional 7,776 instances.

All instances consider an infinite horizon setting and the long-run average cost objective. A parallel study in the finite horizon setting is being prepared and will be reported in in the near future. As discussed in section 7, in infinite horizon models under the long-run average cost objective, the best lower bound is obtained by setting $\lambda = 0$ and variable shipment and ordering costs may be ignored. All instances use linear holding and backlogging costs at the depot and the retailers, and we limit our instances to those in which storage space at the depot is ample. The lower bound is easily computable using the algorithm described in Zheng and Federgruen (1991). Demand distributions are taken to be Normal. The cost performance of the proposed heuristic strategy is assessed via Monte Carlo simulations over 5,000 periods, by solving an instance of the problem (κA) or (MA) in each period. For each problem instance, we ran one set of simulations using (MA) and one set using (κA), and picked the lowest cost. We picked κ using the first method described in Section 6. Almost invariably, the (κA) allocation heuristic outperformed (MA). At the end of this section, we provide a detailed comparison between the performance of the two heuristics on an example instance.

8.1 Numerical Study I

Table 1 summarizes the parameters and meta-parameters that are used to generate the various problem instances in our first numerical study. The remaining parameters are generated as follows:

Cost Ratios

We test three configurations for the holding and backorder cost rates at each retailer:

Uniform costs : $h_j = 1$ and $p_j = 10$, for all $j = 1, \cdots, J$.

Proportional costs : Individual backorder cost rates are selected uniformly in the interval $[8, 12]$. The holding cost rates are then specified to ensure that $\frac{p_j}{h_j} = 10$ for each j, i.e. $h_j = \frac{p_j}{10}$ for all $j = 1, \cdots, j$.

Random costs : Individual backorder costs rates are, again, selected uniformly from the interval $[8, 12]$. Individual holding cost rates are chosen uniformly and independently from the interval $[0.5, 3]$.

The holding cost rate at the depot is set to `overPenalty` times the smallest of the holding cost

Parameter	Values	
J	9	The number of retailers. A value of 9 gives a distribution network with 10 locations.
K	80	The fixed cost of orders by the depot from the supplier.
α	5%	The target likelihood of an overflow of inventory at any of the retailers in any period.
L	1, 3	The leadtime faced by the depot when ordering from the supplier.
ℓ	1, 2	The leadtime faced by every retailer when receiving shipments from the depot.
`overPenalty`	0.5, 1	A 'meta-parameter' specifying the ratio of the holding cost rate at the depot to the *smallest* of the holding rates at the retailers.
`chiBase`	0, 1, 2, 5, 7, 999	A 'meta-parameter' used to specify the capacity at each retailer.
`meanBase`	1, 5, 10	A 'meta-parameter' used the specify the mean single-period demand at each retailer.
`CVBase`	0.15, 0.3, 0.4	A 'meta-parameter' used the specify the coefficient of variation of the single-period demand at each retailer.

Table 1: Parameters used in Numerical Study I. Note that `overPenalty`, `chiBase`, `meanBase`, and `CVBase` are 'meta-parameters' used to generate the remaining parameters in the model. See the body of the paper for more details.

rates at the retailers:

$$H = \texttt{overPenalty} \cdot \min_j h_j$$

It is well-known that holding cost rates *increase* as we progress from one echelon in a supply network to the next. A large, often dominant, part of the holding costs consists of capital costs. Similarly, large depots are typically located where rents and other real-estate costs are lower; unit costs at the depot are additionally reduced by virtue of economies of scale. It is therefore without loss of generality that our study limits the `overPenalty` parameter to values at or below 1.

Retailers' Capacities

We test two configurations for the capacities at each of the retailers:

Equal fractile capacities : $x_j = \mu_j + \texttt{chiBase} \cdot \sigma_j$ for all $j = 1, \cdots, J$.

Random capacities : We first specify that the *total* storage capacity across all retailers, χ_+, should be given by $\chi_+ = \mu_+ + \texttt{chiBase} \cdot \sigma_+$.

The allocation of this total storage capacity χ_+ among the retailers is done randomly, ensuring that each capacity value is between 50% and 150% of the average.

Demand Distributions

As mentioned, all one-period demand distributions are $N(\mu_j, \sigma_j)$ Normals. We test three configurations for the parameters μ_j and σ_j at each retailer j:

Uniform demands : $\mu_j = \texttt{meanBase}$ and $\sigma_j = \texttt{CVBase} \cdot \mu_j$ for all $j = 1, \cdots, J$.

Constant coefficients of variation : The means $\{\mu_j\}$ are selected randomly so as to ensure that $\mu_+ = J \cdot \texttt{meanBase}$; moreover, the randomization procedure ensures that $min_j \mu_j \geq \frac{1}{2}\texttt{meanBase}$ and $\max_j \mu_j \leq 1.5 \cdot \texttt{meanBase}$. Thus, the ratio between the largest and smallest retailer is at most 3. To ensure equal coefficients of variation, we set $\sigma_j = \texttt{CVBase} \cdot \mu_j$ for $j = 1, \cdots, J$.

Random coefficients of variation : The means $\{\mu_j\}$ are generated as under the "Constant coefficients of variation" procedure. The retailers' coefficients of variation $\mathrm{CV}_J = \sigma_j/\mu_j$ are selected uniformly from the interval $[0.5, \texttt{CVBase}]$.

Numerical Results

In tables below, we report on the measured bounds $(UB - LB)/UB$, where UB denotes the simulated cost value of the proposed strategy, and LB the above described lower bound. Note that this ratio represents a *conservative* upper bound for the true optimality gap of the proposed strategy, as well as a conservative upper bound for the accuracy gap of the lower bound approximate LB. Within parentheses, we report on the expected number of periods between consecutive orders from the supplier, i.e. $M(S^\star, s^\star)$ (see equation (26)), with $(s^\star, S^\star)$ the policy parameters of the (s, S) policy that optimizes this Lower Bound DP which is used as part of the proposed strategy. We have organized the results in 48 tables, each reporting on 162 instances. Each of the 48 tables corresponds to a specific choice of (i) L; (ii) ℓ; (iii) the `overPenalty` parameter; (iv) the cost coefficient pattern; and (v) the capacity assignment pattern.

Within each table, there are 6 column groups, one for each value of `chiBase`, and each group comprises 3 columns, one for each value of `CVBase`. Similarly, there are three row groups, one for each value of `meanBase`, each of which is divided into three rows, one for each of the patterns by which the demand distributions are generated.

In Tables 2-5, we display four of the 48 tables. (The remaining 44 tables are provided in an online appendix).

On average, calculation of the lower bound took 8.96 seconds, and in 95% of the instances took less than 22.78 seconds. Calculation of the upper bound with Monte Carlo simulations over 5,000 periods took 6.85 seconds on average, with 95% of the instances running in under 8.76 seconds.

These CPU times reflect an implementation on a machine with two quad-core 2.5 GHz AMD Opteron 2380 processors.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.28 (6)	0.55 (6)	0.52 (6)	0.26 (6)	0.54 (6)	0.61 (7)	0.17 (6)	0.5 (6)	0.6 (6)	≤ 0 (6)	0.17 (6)	0.58 (7)	≤ 0 (6)	0.09 (7)	0.53 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)
	Const. CV	0.45 (6)	0.67 (6)	0.62 (6)	0.57 (6)	0.8 (7)	0.82 (6)	0.71 (6)	1.03 (7)	1.09 (7)	0.57 (6)	1.19 (7)	1.82 (6)	0.36 (6)	0.7 (7)	1.1 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)
	Rand. CV	≤ 0 (6)	≤ 0 (6)	0.01 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	0.01 (6)	≤ 0 (6)	0.06 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	0.4 (2)	0.54 (2)	0.52 (2)	0.41 (2)	0.57 (2)	0.57 (2)	0.32 (2)	0.55 (2)	0.56 (2)	0.08 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.61 (2)	0.68 (2)	0.63 (2)	0.79 (2)	0.91 (2)	0.83 (2)	0.99 (2)	1.19 (2)	1.1 (2)	0.98 (2)	1.26 (2)	1.44 (3)	0.74 (2)	0.44 (2)	0.6 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	0.02 (2)	0.03 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.44 (1)	0.53 (1)	0.49 (1)	0.45 (1)	0.58 (1)	0.54 (1)	0.34 (1)	0.53 (1)	0.53 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.65 (1)	0.67 (1)	0.63 (1)	0.84 (1)	0.9 (1)	0.84 (1)	1.05 (1)	1.18 (1)	1.16 (1)	1.04 (1)	1.16 (1)	1.49 (2)	0.79 (1)	0.46 (1)	0.49 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0 (1)	0.01 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	0.09 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 2: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.52 (7)	0.67 (8)	0.68 (8)	0.65 (7)	0.9 (8)	0.87 (8)	0.78 (7)	1.17 (8)	1.22 (8)	0.68 (8)	1.34 (8)	2.06 (8)	0.53 (7)	0.85 (8)	1.72 (9)	≤ 0 (8)	≤ 0 (8)	0.34 (8)
	Const. CV	≤ 0 (8)	0.19 (8)	0.25 (8)	≤ 0 (8)	0.12 (8)	0.14 (8)	≤ 0 (7)	0 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (9)	0.34 (8)	≤ 0 (8)	≤ 0 (9)	0.5 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)
	Rand. CV	0.28 (7)	0.26 (7)	0.3 (8)	0.26 (7)	0.31 (8)	0.33 (7)	0.3 (7)	0.37 (7)	0.47 (8)	0.22 (8)	0.11 (8)	0.22 (8)	0.11 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.68 (3)	0.71 (3)	0.65 (3)	0.87 (3)	0.97 (3)	0.9 (3)	1.1 (3)	1.29 (3)	1.25 (3)	1.04 (3)	1.45 (3)	1.83 (3)	0.87 (3)	0.98 (3)	1.32 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	0.02 (3)	0.2 (3)	0.16 (3)	≤ 0 (3)	0.11 (3)	0.07 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	0.31 (3)	0.35 (3)	0.35 (3)	0.35 (3)	0.52 (3)	0.51 (3)	0.34 (3)	0.58 (3)	0.65 (3)	0.3 (3)	0.37 (3)	0.41 (3)	0.12 (3)	0.06 (3)	0.03 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.7 (2)	0.71 (2)	0.64 (2)	0.93 (2)	0.99 (2)	0.89 (2)	1.15 (2)	1.34 (2)	1.26 (2)	1.2 (2)	1.54 (2)	1.65 (2)	1.05 (2)	0.83 (2)	0.9 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.05 (2)	0.17 (2)	0.14 (2)	≤ 0 (2)	0.05 (2)	0.03 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.04 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.34 (2)	0.37 (2)	0.37 (2)	0.38 (2)	0.51 (2)	0.54 (2)	0.38 (2)	0.62 (2)	0.68 (2)	0.34 (2)	0.39 (2)	0.37 (2)	0.15 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 3: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.38 (7)	0.56 (8)	0.53 (8)	0.5 (7)	0.82 (8)	0.81 (8)	0.55 (8)	1.09 (8)	1.19 (8)	0.4 (8)	0.83 (8)	1.07 (8)	0.2 (7)	0.47 (8)	0.85 (9)	≤ 0 (8)	≤ 0 (9)	0.55 (8)
	Const. CV	≤ 0 (7)	0.14 (8)	0.16 (8)	≤ 0 (7)	0.03 (8)	0.09 (8)	≤ 0 (7)	≤ 0 (8)	0.11 (8)	≤ 0 (7)	≤ 0 (8)	0.54 (8)	≤ 0 (8)	≤ 0 (8)	0.31 (8)	≤ 0 (8)	0.07 (8)	0.36 (8)
	Rand. CV	0.25 (8)	0.18 (7)	0.23 (7)	0.21 (7)	0.23 (7)	0.35 (7)	0.27 (8)	0.21 (7)	0.33 (7)	0.16 (8)	≤ 0 (8)	0.08 (8)	0.05 (7)	≤ 0 (8)	≤ 0 (9)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.61 (3)	0.62 (3)	0.56 (3)	0.8 (3)	0.99 (3)	0.88 (3)	0.86 (3)	1.27 (3)	1.28 (3)	0.81 (3)	0.84 (3)	0.83 (3)	0.62 (3)	0.34 (3)	0.2 (3)	≤ 0 (3)	≤ 0 (3)	0.08 (3)
	Const. CV	≤ 0 (3)	0.11 (3)	0.09 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	0.06 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.22 (3)	0.33 (3)	0.32 (3)	0.26 (3)	0.44 (3)	0.49 (3)	0.27 (3)	0.47 (3)	0.52 (3)	0.19 (3)	0.12 (3)	0.11 (3)	0.06 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.64 (2)	0.67 (2)	0.56 (2)	0.83 (2)	1.01 (2)	0.92 (2)	0.88 (2)	1.29 (2)	1.31 (2)	0.81 (2)	0.94 (2)	0.85 (2)	0.61 (2)	0.52 (2)	0.13 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (2)	0.15 (2)	0.08 (2)	≤ 0 (2)	0.02 (2)	≤ 0 (2)	0.09 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.25 (2)	0.33 (2)	0.34 (2)	0.29 (2)	0.45 (2)	0.53 (2)	0.32 (2)	0.54 (2)	0.57 (2)	0.22 (2)	0.21 (2)	0.15 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 4: Results for $L = 1$, $\ell = 1$, `overPenalty = 0.5`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.41 (6)	0.66 (5)	0.62 (6)	0.52 (6)	0.77 (6)	0.75 (6)	0.64 (5)	0.99 (6)	1.1 (6)	0.67 (6)	2.73 (6)	4.19 (6)	2.43 (5)	7.51 (6)	9.66 (6)	77.08 (5)	61.71 (6)	52.43 (6)
	Const. CV	≤ 0 (5)	0.18 (6)	0.24 (6)	≤ 0 (5)	0.13 (6)	0.07 (6)	≤ 0 (6)	0.07 (6)	0.14 (6)	0.07 (5)	1.35 (6)	2.1 (6)	0.59 (6)	4.17 (6)	5.53 (6)	73.9 (5)	58.1 (6)	56.34 (6)
	Rand. CV	0.19 (5)	0.14 (6)	0.2 (6)	0.23 (5)	0.19 (6)	0.27 (5)	0.19 (5)	0.19 (6)	0.35 (5)	0.14 (5)	0.53 (5)	0.73 (6)	0.63 (5)	1.62 (6)	3.27 (5)	76.17 (5)	74.14 (5)	67.76 (6)
meanBase = 5	Uniform Demands	0.6 (2)	0.66 (2)	0.61 (2)	0.78 (2)	0.88 (2)	0.82 (2)	0.98 (2)	1.18 (2)	1.17 (2)	1.1 (2)	2.91 (2)	5.07 (2)	3.17 (2)	15.35 (2)	19.4 (2)	69.67 (2)	54.13 (2)	24.99 (3)
	Const. CV	0 (2)	0.15 (2)	0.12 (2)	≤ 0 (2)	0.04 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.1 (2)	0.78 (2)	1.43 (2)	1.19 (2)	7.19 (2)	10.4 (3)	66.18 (2)	50.58 (2)	24.54 (3)
	Rand. CV	0.26 (2)	0.31 (2)	0.33 (2)	0.3 (2)	0.46 (2)	0.47 (2)	0.29 (2)	0.46 (2)	0.58 (2)	0.25 (2)	0.66 (2)	1.17 (2)	0.61 (2)	2.94 (2)	5.21 (2)	71.97 (2)	66.83 (2)	63.28 (2)
meanBase = 10	Uniform Demands	0.63 (1)	0.63 (1)	0.57 (1)	0.83 (1)	0.87 (1)	0.82 (1)	1.03 (1)	1.19 (1)	1.16 (1)	1.16 (1)	2.65 (1)	4.29 (2)	2.91 (1)	18.13 (1)	25.22 (2)	54.53 (1)	16.13 (2)	12.64 (2)
	Const. CV	0.03 (1)	0.17 (1)	0.09 (1)	≤ 0 (1)	0.05 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.41 (1)	1.13 (2)	1.13 (1)	6.69 (1)	13.25 (2)	50.87 (1)	15.35 (2)	12.13 (2)
	Rand. CV	0.29 (1)	0.31 (1)	0.31 (1)	0.33 (1)	0.43 (1)	0.44 (1)	0.33 (1)	0.49 (1)	0.56 (1)	0.29 (1)	0.59 (1)	0.99 (1)	0.6 (1)	3.63 (1)	6.08 (1)	61.88 (1)	52.29 (1)	45.96 (1)

Table 5: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

Considering tables 2-5, one notices that the optimality gaps are invariably very small, and below 2%. The gaps tend to increase with the `CVBase` parameter, as may be expected. The larger the demand variability, the more frequently each retailer encounters unexpected imbalances in the inventory levels, which in the relaxed DP can be addressed with return shipments, and in the real system cannot.

The excellent performance of the heuristic and excellent accuracy of the lower bound is consistent irrespective of the length of the expected cycle time between orders. The latter may be as low as 2 or as high as 9; (the latter when `meanBase` equals 1, and under random costs and random capacities; see tables 3 and 4). One might have anticipated performance to deteriorate as the cycle length increases and the opportunities to redress imbalances with new inventory at the depot arise less frequently.

The results in table 5, with `overPenalty`=1, are more mixed. A significant optimality gap arises when the storage capacities at the retailers become large in the last two columns groups. This deterioration in our heuristic is explained by the fact that when it is equally expensive to store items at the depot as at the retailers, and, in addition, retailers have large storage capacities, it is optimal in the *relaxed* DP to assign all of the system-wide inventory at one or several of the retailers, since in the relaxed DP it is optimal to allocate inventories myopically. In the relaxed DP, future imbalances may be redressed via return shipments to the depot; this is not possible in the real system. However, when the retailer storage capacities are modest, a significant part of the system-wide inventory will remain at the depot both in the real system and in the relaxed DP, and this inventory can be used to redress any future imbalances even when no new orders arrive from the external supplier.

We therefore conclude that our relaxation bound approach performs excellently across virtually all parameter combinations. The exception is settings in which the holding cost at the depot is unusually large, and the retailer storage capacities are large.

Given the sensitivity to the `overPenalty` parameter, we construct a second numerical study, in which this parameter is varied from 0.25 to 1 in increments of 0.25. If our above explanation for the performance difference is correct, we anticipate excellent performance for all systems in which it is cheaper to store inventories at the depot.

With sufficient inventory kept at the depot, retailers can be replenished in every period and it is therefore reasonable to set their inventory capacity at an appropriate fractile of their *single-period* demand distribution. However, sometimes only a small amount of inventory is held at the depot, in which case retailers get replenished less frequently. We therefore extend our second numerical study to assess performance of the bounds when inventory capacities are based on 4-period demand distributions. (The number 4 is chosen as the average of the replenishment cycle lengths in the scenarios considered in the first study; see tables 2-5 for examples).

We conclude this Section with a detailed comparison of the performance of the (κA) and the Myopic Allocation (MA) heuristics, in a given example instance. We again consider a system with $J = 10$ retailers, in this case all with an identical Normal distribution $N(1, 0.3^2)$ for their single-period demands. Demands are assumed to be independent across the 10 retailers, so that system-wide demand in any given period is $N(10, 0.95^2)$. All retailers have an identical storage capacity of $\chi_j = 10$ units, and an identical backorder-to-holding cost ratio; specifically, $p_j/h_j = 10$. The holding cost values $\{h_j : j = 1, \cdots, 10\}$ were generated from a distribution with mean 1, but vary significantly (between $h_8 = 0.82$ and $h_4 = 1.11$). The holding cost value at the depot is relatively high, with $H = 10$. As argued above, this is highly uncommon in practical applications, but the choice was made to create an instance in which keeping central inventories at the depot is uneconomical. The major cost savings achieved by the (κA)-allocation strategy are therefore entirely due to a superior allocation of incoming orders among the retailers. Under more realistic holding cost rates at the depot, the (κA)-allocation strategy displays additional advantages in keeping some of the system inventory back at the depot, thereby enabling a centrally available common safety stock.

Fixed and variable procurement costs are such that in the relaxed DP, an (s, S) policy is optimal with $s = 69$ and $S = 109$. This implies that the average cycle between consecutive orders is 4.98 periods, with a standard deviation of only 0.14 periods. When simple myopic allocations (MA) are used, the resulting long run average cost upper bound is 94.75, with an optimality gap of 61%. In contrast, when the (κA)-allocation strategy is used, the long-run average cost upper bound is reduced to 41.02, leaving an (upper bound) for the optimality gap of less than 10%. This is still considerably larger than what we observe in our numerical studies, but this is due to the unrealistically high relative storage costs at the depot, as per the discussion above.

Figure 1 displays inventory and backlogging plots for each of the 10 retailers. The black dots represent the end-of-period inventory levels observed during the simulations at each of the retailers, followed by a histogram thereof (solid line). One notes that under Myopic Allocations (MA), the retailers with relatively cheap holding costs get an abundance of inventory allocated whereas those with relatively expensive holding costs get insufficient inventories and, as a consequence, incur

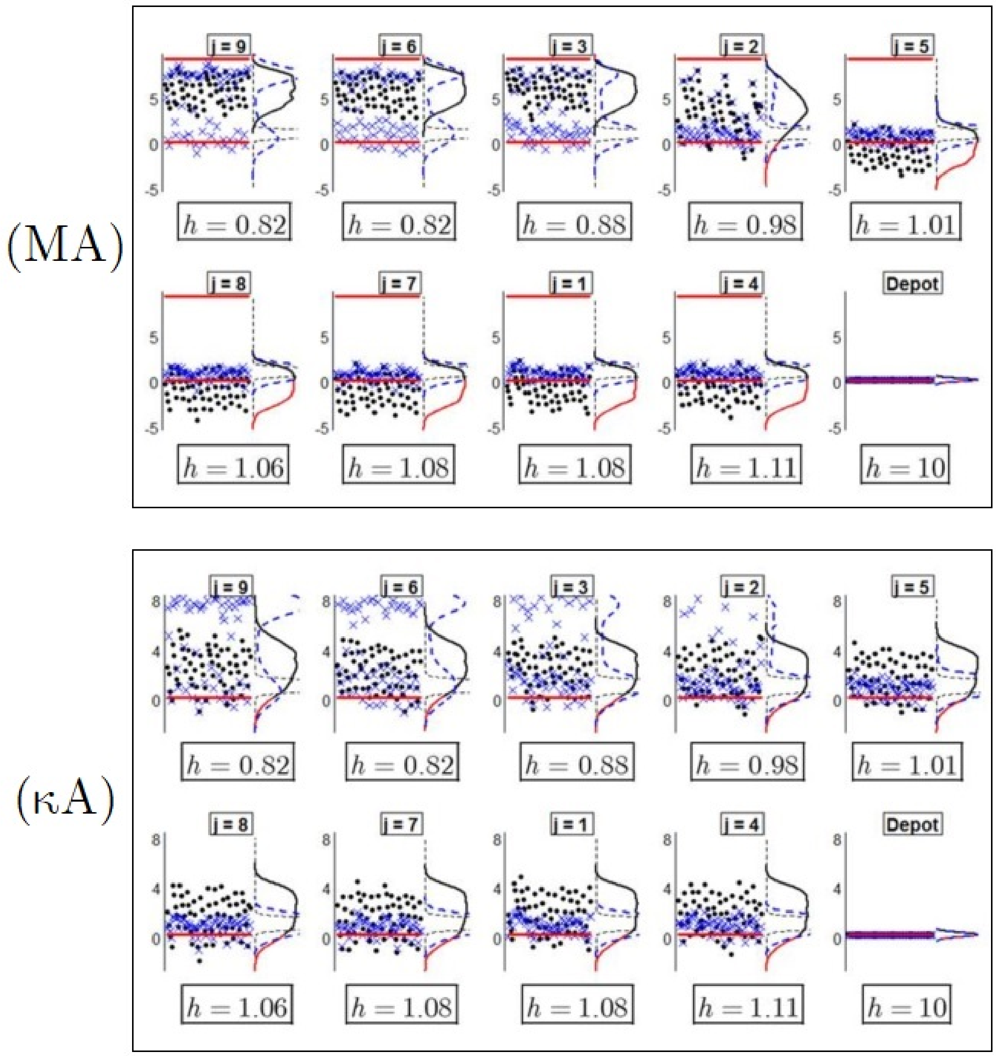

Figure 1: System simulations using Myopic Allocation Policies (top) and (κA) Allocation Policies (bottom). The black dots represent end-of-period inventory levels observed during the simulations at each of the retailers (followed by a histogram thereof as solid lines). The blue crosses represent the end-of-period inventories that would be attained in a *hypothetical* setting in which the non-negativity constraints on shipments are relaxed during the simulation (followed by a histogram thereof as a dotted line). These crosses, in some sense, represent the strategy that would lead to the lower bound value.

backlogs in later periods. (This is because the Myopic Allocation policy only plans one period ahead, where it is indeed optimal to send most of the inventory to retailers with cheap holding costs.) The (κA) allocation strategy, in contrast, reduces allocations to the former and expands those to the latter, thus simultaneously reducing inventory levels and backlogging sizes in the system.

8.2 Numerical Study II

Our second study consists of 7,776 problem instances, also arranged in 48 tables. The objective of the second study is to investigate to what extent the performance of the lower and upper bounds are affected by

(a) the value of the `overPenalty`, i.e. how much cheaper it is to store inventory at the depot than at the retailers.

(b) the relationship between the retailers' inventory capacity and the characteristics of their demand distributions; in addition to specifying the capacity as a given number (`chiBase`) of standard deviations above the mean of their *single* period demand, we also generate capacities with respect to the distribution of the *4-period* demands at each retailer.

Our two numerical studies share 1,944 instances, so that the total number of distinct instances over these two studies is 13,688.

The design of the second numerical study is identical to that of study I, except that we restrict ourselves to a single pair of leadtime values: $L = 3$ and $\ell = 2$. However, we consider 4 values for the `overPenalty` parameter; 0.25, 0.5, 0.75 and 1.

Moreover, we consider four ways to specify the retailer capacities:

(1) Equal fractiles, based on single period demand distributions as above.

(2) Random capacities, based on single period demand distributions, as above.

(3) Equal fractiles, based on 4-period demand distributions:

$$\chi_j = 4\mu_j + 2\texttt{chiBase} \cdot \sigma_j, \quad j = 1, \cdots, J$$

(4) Random capacities, based on 4-period demand distributions. We first specify $\chi_+ = 4\mu_+ + 2\texttt{chiBase} \cdot \sigma_+$. The allocation of the total storage capacity χ_+ among the retailers is then done randomly, ensuring that each capacity value is between 50% and 150% of the average.

In Appendix **??**, we display 4 of the 48 tables in this numerical study. These tables display uniformly excellent performances as long as it is cheaper to store goods at the depot than at the retailers. With retailers' capacities set with respect to 4 period demands, significant optimality gaps never appear, irrespective of the value of `chiBase`.

9 Conclusion

We have devised a solution approach for replenishment strategies in a general two-echelon distribution system, under periodic review. Inventories may be kept at any of the facilities, but each facility has a limited storage capacity. Our model allows for arbitrary joint distributions of each period's demands among the retailers.

The solution approach generates a dynamic order and allocation policy based on an (s, S)-type policy that is optimal in a lower bound DP. The lower bound is obtained as the Lagrangian dual associated with the exact DP, which, itself, is intractable. However, we have shown that this Lagrangian dual can be solved very efficiently. Our approach generates both a lower and an upper bound for the optimal cost value and, hence, a conservative bound for the optimality gap of the proposed strategy and the accuracy gap of the lower bound. An extensive numerical study with about 14,000 problem instances reveals that the lower bound and the proposed replenishment strategy perform excellently, almost across the entire parameter space.

Our results were derived under a number of standard assumptions about the shape of the cost functions; order costs have a fixed and a variable component, while inventory carrying and backlogging costs are convex functions of the end-of-period inventory level and backlog level, respectively. However, as described in the Introduction, in the absence of these structural properties, our main results continue to prevail: the Lagrangian relaxed DP can still be reduced to a single DP with a one-dimensional state space. While the optimal order strategy in the lower bound DP, in general, fails of be of a simple (s, S) structure, it can be computed efficiently and paired with any of the above heuristic allocation policies. The only complication, in this general setting, is that the allocation mathematical programs (MA) and (κA), in section 6, are no longer convex programs so that more general, and less efficient solution methods would be needed. Finally, the derivative expressions in Lemma 1 may no longer apply, requiring an adaptation of the supergradient result provided in Theorem 3.

As noted in Federgruen and Zipkin (1984b) for the systems considered there, the ability to compute an accurate lower bound on the optimal cost for any set of parameters has fundamental benefits beyond the challenge of constructing effective replenishment strategies. These bounds allow us to address various strategic questions; for example, to evaluate what impact the two lead times L and ℓ have on the overall system-wide performance, an important input when selecting suppliers and distribution centers. (Our methods are readily extended when the leadtimes are stochastic themselves, as long as they are governed by a so-called exogenous sequential process, see Zipkin (2000)). Similarly, the model can be used to evaluate the benefit of depot inventories, and of investments in storage capacity.

Again, as pointed out by Federgruen and Zipkin (1984b), the same model and solution approach applies when managing a product line of related products, all sold from the same underlying facility and manufactured in two stages; in the first stage, a common intermediate item is produced, with differentiating features and accessories added in a second phase.

Future work should address a model capable of handling an arbitrary number of items sold from a general network of sales locations.

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A Results for Numerical Study I

This appendix contains all 48 tables of results for our first numerical study, *including* the four tables provided in the paper.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.15 (5)	0.45 (6)	0.4 (5)	0.12 (5)	0.38 (5)	0.48 (6)	≤ 0 (5)	0.24 (5)	0.4 (6)	≤ 0 (5)	0.1 (6)	0.52 (6)	≤ 0 (5)	0.1 (6)	0.52 (6)	≤ 0 (5)	0.1 (6)	0.52 (6)
	Const. CV	0.23 (5)	0.53 (6)	0.51 (5)	0.18 (5)	0.54 (5)	0.62 (6)	≤ 0 (5)	0.33 (5)	0.44 (6)	≤ 0 (5)	≤ 0 (6)	0.55 (6)	≤ 0 (5)	≤ 0 (6)	0.55 (6)	≤ 0 (5)	≤ 0 (6)	0.55 (6)
	Rand. CV	0.21 (5)	0.22 (5)	0.35 (5)	0.14 (5)	0.14 (5)	0.26 (5)	0.07 (5)	≤ 0 (5)	0.02 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)
meanBase = 5	Uniform Demands	0.39 (2)	0.49 (2)	0.43 (2)	0.39 (2)	0.55 (2)	0.49 (2)	0.18 (2)	0.35 (2)	0.36 (2)	≤ 0 (2)	≤ 0 (2)	0.04 (2)	≤ 0 (2)	≤ 0 (2)	0.04 (2)	≤ 0 (2)	≤ 0 (2)	0.04 (2)
	Const. CV	0.49 (2)	0.58 (2)	0.51 (2)	0.49 (2)	0.62 (2)	0.54 (2)	0.17 (2)	0.32 (2)	0.29 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.43 (2)	0.51 (2)	0.55 (2)	0.38 (2)	0.48 (2)	0.54 (2)	0.09 (2)	0.12 (2)	0.15 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.5 (1)	0.51 (1)	0.46 (1)	0.55 (1)	0.58 (1)	0.53 (1)	0.31 (1)	0.38 (1)	0.27 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.6 (1)	0.61 (1)	0.56 (1)	0.63 (1)	0.64 (1)	0.61 (1)	0.29 (1)	0.3 (1)	0.23 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.52 (1)	0.55 (1)	0.58 (1)	0.5 (1)	0.57 (1)	0.55 (1)	0.18 (1)	0.19 (1)	0.18 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 6: Results for $L = 1$, $\ell = 1$, `overPenalty = 0.5`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (5)	0.34 (6)	0.32 (5)	≤ 0 (5)	0.16 (6)	0.23 (6)	≤ 0 (5)	0.06 (5)	0.14 (6)	≤ 0 (5)	0.06 (6)	0.33 (6)	≤ 0 (5)	≤ 0 (6)	0.55 (6)	≤ 0 (5)	0.1 (6)	0.52 (6)
	Const. CV	0.13 (5)	0.45 (5)	0.53 (6)	0.08 (5)	0.44 (5)	0.49 (6)	0.08 (5)	0.31 (6)	0.61 (6)	≤ 0 (5)	≤ 0 (6)	0.55 (6)	≤ 0 (5)	≤ 0 (6)	0.36 (6)	≤ 0 (5)	≤ 0 (6)	0.55 (6)
	Rand. CV	0.24 (5)	0.28 (5)	0.39 (5)	0.3 (5)	0.3 (5)	0.42 (6)	0.35 (5)	0.34 (5)	0.46 (6)	0.27 (5)	0.1 (5)	0.28 (5)	0.13 (5)	0.01 (5)	0.02 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)
meanBase = 5	Uniform Demands	0.09 (2)	0.38 (2)	0.37 (2)	≤ 0 (2)	0.12 (2)	0.17 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.04 (2)
	Const. CV	0.3 (2)	0.5 (2)	0.47 (2)	0.25 (2)	0.5 (2)	0.45 (2)	0.21 (2)	0.37 (2)	0.51 (2)	0.09 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.37 (2)	0.51 (2)	0.58 (2)	0.43 (2)	0.56 (2)	0.67 (2)	0.48 (2)	0.64 (2)	0.73 (2)	0.43 (2)	0.38 (2)	0.36 (2)	0.29 (2)	0.2 (2)	0.18 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.14 (1)	0.36 (1)	0.36 (1)	≤ 0 (1)	0.11 (1)	0.1 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.39 (1)	0.52 (1)	0.51 (1)	0.36 (1)	0.48 (1)	0.43 (1)	0.29 (1)	0.39 (1)	0.37 (1)	0.14 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.45 (1)	0.55 (1)	0.6 (1)	0.53 (1)	0.65 (1)	0.68 (1)	0.6 (1)	0.76 (1)	0.81 (1)	0.57 (1)	0.51 (1)	0.48 (1)	0.38 (1)	0.24 (1)	0.17 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 7: Results for $L = 1$, $\ell = 1$, `overPenalty = 0.5`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.24 (6)	0.45 (7)	0.49 (6)	0.17 (6)	0.53 (6)	0.57 (6)	≤ 0 (6)	0.34 (6)	0.34 (6)	≤ 0 (6)	0.21 (7)	0.64 (7)	≤ 0 (6)	0.21 (7)	0.64 (7)	≤ 0 (6)	0.21 (7)	0.64 (7)
	Const. CV	0.24 (6)	0.45 (7)	0.43 (6)	0.18 (6)	0.48 (6)	0.46 (6)	≤ 0 (6)	0.26 (6)	0.21 (6)	≤ 0 (6)	0.05 (7)	0.37 (7)	≤ 0 (6)	0.05 (7)	0.37 (7)	≤ 0 (6)	0.05 (7)	0.37 (7)
	Rand. CV	0.11 (6)	0.01 (6)	0.02 (6)	0.15 (6)	≤ 0 (6)	≤ 0 (6)	0.02 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)
meanBase = 5	Uniform Demands	0.44 (2)	0.54 (2)	0.5 (2)	0.44 (2)	0.59 (2)	0.56 (2)	0.18 (2)	0.29 (2)	0.34 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Const. CV	0.45 (2)	0.53 (2)	0.51 (2)	0.45 (2)	0.55 (2)	0.52 (2)	0.16 (2)	0.23 (2)	0.26 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.17 (2)	0.16 (2)	0.12 (2)	0.13 (2)	0.11 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.54 (1)	0.56 (1)	0.48 (1)	0.59 (1)	0.63 (1)	0.54 (1)	0.34 (1)	0.36 (1)	0.28 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.54 (1)	0.58 (1)	0.5 (1)	0.57 (1)	0.63 (1)	0.56 (1)	0.28 (1)	0.33 (1)	0.27 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.26 (1)	0.17 (1)	0.15 (1)	0.22 (1)	0.11 (1)	0.09 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 8: Results for $L = 1$, $\ell = 1$, `overPenalty = 0.5`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.14 (6)	0.47 (6)	0.45 (6)	0.08 (6)	0.37 (7)	0.41 (6)	0.04 (6)	0.47 (6)	0.44 (6)	≤ 0 (7)	0.04 (7)	0.68 (7)	≤ 0 (6)	0.07 (7)	0.48 (7)	≤ 0 (6)	0.21 (7)	0.64 (7)
	Const. CV	0.34 (6)	0.55 (6)	0.51 (7)	0.4 (6)	0.77 (6)	0.67 (7)	0.43 (6)	0.91 (7)	0.97 (7)	0.26 (6)	0.57 (6)	0.84 (7)	0.05 (6)	0.22 (7)	0.43 (7)	≤ 0 (6)	0.05 (7)	0.37 (7)
	Rand. CV	0.01 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	0.06 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)
meanBase = 5	Uniform Demands	0.28 (2)	0.46 (2)	0.46 (2)	0.24 (2)	0.42 (2)	0.44 (2)	0.21 (2)	0.39 (2)	0.42 (2)	0.09 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Const. CV	0.52 (2)	0.58 (2)	0.57 (2)	0.67 (2)	0.83 (2)	0.82 (2)	0.75 (2)	1.12 (2)	1.16 (2)	0.61 (2)	0.58 (2)	0.64 (2)	0.34 (2)	0.1 (2)	0.11 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.36 (1)	0.47 (1)	0.43 (1)	0.31 (1)	0.46 (1)	0.38 (1)	0.26 (1)	0.41 (1)	0.36 (1)	0.13 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.6 (1)	0.64 (1)	0.57 (1)	0.78 (1)	0.93 (1)	0.87 (1)	0.91 (1)	1.19 (1)	1.19 (1)	0.82 (1)	0.75 (1)	0.65 (1)	0.46 (1)	0.12 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.02 (1)	≤ 0 (1)	≤ 0 (1)	0.08 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 9: Results for $L = 1$, $\ell = 1$, `overPenalty = 0.5`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.28 (7)	0.5 (8)	0.47 (8)	0.23 (7)	0.58 (8)	0.55 (8)	≤ 0 (7)	0.37 (9)	0.42 (8)	≤ 0 (8)	≤ 0 (9)	0.55 (8)	≤ 0 (8)	≤ 0 (9)	0.55 (8)	≤ 0 (8)	≤ 0 (9)	0.55 (8)
	Const. CV	≤ 0 (7)	0.2 (8)	0.2 (8)	≤ 0 (7)	0.21 (8)	0.02 (8)	≤ 0 (7)	0.05 (9)	0.07 (8)	≤ 0 (8)	0.07 (8)	0.36 (8)	≤ 0 (8)	0.07 (8)	0.36 (8)	≤ 0 (8)	0.07 (8)	0.36 (8)
	Rand. CV	0.13 (7)	0.1 (7)	0.19 (7)	0.09 (7)	0.07 (7)	0.16 (8)	0.09 (7)	≤ 0 (7)	0.02 (7)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.49 (3)	0.59 (3)	0.49 (3)	0.51 (3)	0.68 (3)	0.56 (3)	0.34 (3)	0.45 (3)	0.34 (3)	≤ 0 (3)	≤ 0 (3)	0.08 (3)	≤ 0 (3)	≤ 0 (3)	0.08 (3)	≤ 0 (3)	≤ 0 (3)	0.08 (3)
	Const. CV	0.15 (3)	0.18 (3)	0.1 (3)	0.1 (3)	0.14 (3)	0.08 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.16 (3)	0.27 (3)	0.29 (3)	0.12 (3)	0.27 (3)	0.28 (3)	0.01 (3)	0.07 (3)	0.18 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.54 (2)	0.6 (2)	0.49 (2)	0.59 (2)	0.7 (2)	0.61 (2)	0.32 (2)	0.47 (2)	0.37 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.18 (2)	0.22 (2)	0.12 (2)	0.13 (2)	0.19 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.23 (2)	0.28 (2)	0.31 (2)	0.21 (2)	0.28 (2)	0.36 (2)	≤ 0 (2)	0.13 (2)	0.13 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 10: Results for $L = 1$, $\ell = 1$, `overPenalty = 0.5`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.38 (7)	0.56 (8)	0.53 (8)	0.5 (7)	0.82 (8)	0.81 (8)	0.55 (8)	1.09 (8)	1.19 (8)	0.4 (8)	0.83 (8)	1.07 (8)	0.2 (7)	0.47 (8)	0.85 (9)	≤ 0 (8)	≤ 0 (9)	0.55 (8)
	Const. CV	≤ 0 (7)	0.14 (8)	0.16 (8)	≤ 0 (7)	0.03 (8)	0.09 (8)	≤ 0 (7)	≤ 0 (8)	0.11 (8)	≤ 0 (7)	≤ 0 (8)	0.54 (8)	≤ 0 (8)	≤ 0 (8)	0.31 (8)	≤ 0 (8)	0.07 (8)	0.36 (8)
	Rand. CV	0.25 (8)	0.18 (7)	0.23 (7)	0.21 (7)	0.23 (7)	0.35 (7)	0.27 (8)	0.21 (7)	0.33 (7)	0.16 (8)	≤ 0 (8)	0.08 (8)	0.05 (7)	≤ 0 (8)	≤ 0 (9)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.61 (3)	0.62 (3)	0.56 (3)	0.8 (3)	0.99 (3)	0.88 (3)	0.86 (3)	1.27 (3)	1.28 (3)	0.81 (3)	0.84 (3)	0.83 (3)	0.62 (3)	0.34 (3)	0.2 (3)	≤ 0 (3)	≤ 0 (3)	0.08 (3)
	Const. CV	≤ 0 (3)	0.11 (3)	0.09 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	0.06 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.22 (3)	0.33 (3)	0.32 (3)	0.26 (3)	0.44 (3)	0.49 (3)	0.27 (3)	0.47 (3)	0.52 (3)	0.19 (3)	0.12 (3)	0.11 (3)	0.06 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.64 (2)	0.67 (2)	0.56 (2)	0.83 (2)	1.01 (2)	0.92 (2)	0.88 (2)	1.29 (2)	1.31 (2)	0.81 (2)	0.94 (2)	0.85 (2)	0.61 (2)	0.52 (2)	0.13 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (2)	0.15 (2)	0.08 (2)	≤ 0 (2)	0.02 (2)	≤ 0 (2)	0.09 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.25 (2)	0.33 (2)	0.34 (2)	0.29 (2)	0.45 (2)	0.53 (2)	0.32 (2)	0.54 (2)	0.57 (2)	0.22 (2)	0.21 (2)	0.15 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 11: Results for $L = 1$, $\ell = 1$, `overPenalty = 0.5`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.11 (4)	0.32 (3)	0.31 (4)	0.06 (4)	0.22 (4)	0.33 (4)	≤ 0 (4)	0.06 (4)	0.27 (4)	≤ 0 (4)	0.95 (4)	1.55 (4)	≤ 0 (4)	1.71 (4)	3.24 (4)	2.56 (4)	6.16 (4)	8.25 (4)
	Const. CV	0.17 (4)	0.36 (3)	0.44 (4)	0.12 (4)	0.3 (4)	0.45 (4)	≤ 0 (4)	0.08 (4)	0.4 (4)	≤ 0 (4)	0.98 (4)	1.58 (4)	≤ 0 (4)	1.72 (4)	3.3 (4)	2.48 (4)	6 (4)	8.08 (4)
	Rand. CV	0.16 (4)	0.19 (4)	0.27 (4)	0.26 (3)	0.1 (4)	0.19 (4)	0.09 (3)	≤ 0 (4)	≤ 0 (4)	≤ 0 (3)	≤ 0 (4)	0.07 (4)	≤ 0 (3)	0.7 (4)	4.39 (4)	20.46 (3)	4.16 (4)	3.88 (4)
meanBase = 5	Uniform Demands	0.4 (1)	0.46 (1)	0.39 (1)	0.39 (1)	0.43 (1)	0.43 (1)	0.17 (1)	0.25 (1)	0.24 (1)	≤ 0 (1)	≤ 0 (2)	1.11 (2)	≤ 0 (1)	5.75 (2)	6.73 (2)	3.62 (1)	5.94 (2)	6.73 (2)
	Const. CV	0.47 (1)	0.54 (1)	0.47 (1)	0.46 (1)	0.49 (1)	0.45 (1)	0.17 (1)	0.25 (1)	0.15 (1)	≤ 0 (1)	≤ 0 (2)	0.62 (2)	≤ 0 (1)	5.51 (2)	6.44 (2)	3.48 (1)	5.72 (2)	6.44 (2)
	Rand. CV	0.38 (1)	0.46 (1)	0.48 (1)	0.32 (1)	0.41 (1)	0.44 (1)	0.07 (1)	0.09 (1)	0.09 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.24 (1)	4.37 (1)	21.7 (1)	27.81 (1)	28.09 (1)
meanBase = 10	Uniform Demands	0.49 (1)	0.54 (1)	0.48 (1)	0.53 (1)	0.62 (1)	0.55 (1)	0.3 (1)	0.38 (1)	0.31 (1)	≤ 0 (1)	≤ 0 (1)	1.27 (1)	≤ 0 (1)	7.04 (1)	8.35 (1)	4.83 (1)	7.34 (1)	8.35 (1)
	Const. CV	0.57 (1)	0.63 (1)	0.56 (1)	0.59 (1)	0.7 (1)	0.63 (1)	0.27 (1)	0.34 (1)	0.27 (1)	≤ 0 (1)	≤ 0 (1)	0.9 (1)	≤ 0 (1)	6.5 (1)	7.83 (1)	4.65 (1)	6.83 (1)	7.83 (1)
	Rand. CV	0.48 (1)	0.53 (1)	0.54 (1)	0.45 (1)	0.51 (1)	0.52 (1)	0.14 (1)	0.14 (1)	0.13 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.32 (1)	6.52 (1)	≤ 0 (1)	34.1 (1)	34.14 (1)

Table 12: Results for $L = 1$, $\ell = 1$, `overPenalty = 1`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (3)	0.15 (4)	0.24 (4)	≤ 0 (3)	0 (4)	0.17 (4)	≤ 0 (4)	0 (4)	0.21 (4)	0.02 (4)	5.7 (4)	6.67 (4)	4.77 (4)	9.96 (4)	17.58 (4)	2.56 (4)	6.16 (4)	8.25 (4)
	Const. CV	0.09 (4)	0.34 (3)	0.42 (4)	0.08 (3)	0.24 (4)	0.38 (4)	0.03 (4)	0.4 (4)	0.57 (4)	1 (4)	7.53 (4)	6.91 (4)	4.86 (4)	9.05 (4)	15.51 (4)	2.48 (4)	6 (4)	8.08 (4)
	Rand. CV	0.3 (3)	0.25 (4)	0.33 (4)	0.28 (3)	0.28 (3)	0.33 (3)	0.33 (3)	0.3 (3)	0.39 (4)	0.3 (3)	0.69 (4)	1.88 (4)	0.33 (4)	4.46 (4)	8.7 (4)	20.46 (3)	4.16 (4)	3.88 (4)
meanBase = 5	Uniform Demands	0.1 (1)	0.29 (1)	0.31 (1)	≤ 0 (1)	0.09 (1)	0.11 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.09 (1)	9.92 (1)	20.59 (2)	8.32 (1)	33.7 (1)	27.38 (2)	3.62 (1)	5.94 (2)	6.73 (2)
	Const. CV	0.32 (1)	0.42 (1)	0.43 (1)	0.29 (1)	0.42 (1)	0.32 (1)	0.24 (1)	0.41 (1)	0.45 (1)	0.88 (1)	13.37 (1)	23.01 (2)	13.83 (1)	29.26 (2)	19.31 (2)	3.48 (1)	5.72 (2)	6.44 (2)
	Rand. CV	0.36 (1)	0.48 (1)	0.52 (1)	0.43 (1)	0.54 (1)	0.59 (1)	0.48 (1)	0.63 (1)	0.7 (1)	0.44 (1)	1.02 (1)	2.19 (1)	0.66 (1)	7.72 (1)	14.85 (1)	21.7 (1)	27.81 (1)	28.09 (1)
meanBase = 10	Uniform Demands	0.15 (1)	0.4 (1)	0.39 (1)	≤ 0 (1)	0.15 (1)	0.15 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	11.58 (1)	23.75 (1)	≤ 0 (1)	38.65 (1)	31.36 (1)	4.83 (1)	7.34 (1)	8.35 (1)
	Const. CV	0.37 (1)	0.55 (1)	0.53 (1)	0.34 (1)	0.51 (1)	0.45 (1)	0.32 (1)	0.47 (1)	0.47 (1)	0.16 (1)	15.89 (1)	26.54 (1)	≤ 0 (1)	34.41 (1)	22.85 (1)	4.65 (1)	6.83 (1)	7.83 (1)
	Rand. CV	0.42 (1)	0.54 (1)	0.57 (1)	0.49 (1)	0.6 (1)	0.65 (1)	0.56 (1)	0.71 (1)	0.77 (1)	0.53 (1)	0.49 (1)	0.46 (1)	0.38 (1)	0.29 (1)	0.23 (1)	≤ 0 (1)	34.1 (1)	34.14 (1)

Table 13: Results for $L = 1$, $\ell = 1$, `overPenalty = 1`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.17	0.37	0.5	0.04	0.33	0.42	≤ 0	0.12	0.39	≤ 0	0.57	1.48	≤ 0	1.74	2.67	61.28	25.6	25
		(4)	(5)	(5)	(5)	(4)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)
	Const. CV	0.17	0.35	0.49	0.03	0.35	0.39	≤ 0	0.06	0.19	≤ 0	0.62	1.41	≤ 0	1.76	2.58	61.08	27.83	29.59
		(4)	(5)	(5)	(5)	(4)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)
	Rand. CV	0.12	≤ 0	≤ 0	0.1	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	0.02	≤ 0	0.22	0.02	0.25	1.89	72.23	58.62	49.99
		(4)	(4)	(4)	(4)	(5)	(4)	(5)	(5)	(5)	(4)	(5)	(5)	(4)	(5)	(5)	(4)	(5)	(5)
meanBase = 5	Uniform Demands	0.44	0.47	0.45	0.44	0.48	0.49	0.18	0.26	0.22	≤ 0	≤ 0	0.7	≤ 0	1.19	5.62	59.5	7.99	8.17
		(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(2)	(2)	(1)	(2)	(2)	(1)	(2)	(2)
	Const. CV	0.44	0.5	0.46	0.42	0.53	0.46	0.15	0.21	0.17	≤ 0	≤ 0	0.65	≤ 0	0.81	4.96	60.21	8.7	8.45
		(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(2)	(2)	(1)	(2)	(2)	(1)	(2)	(2)
	Rand. CV	0.19	0.13	0.11	0.14	0.06	0.03	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	0.02	1.08	10.62	59.9	54.32
		(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(2)	(1)	(1)	(2)	(1)	(1)	(2)	(1)	(1)
meanBase = 10	Uniform Demands	0.5	0.53	0.48	0.52	0.57	0.52	0.24	0.29	0.21	≤ 0	≤ 0	0.63	≤ 0	1.36	6.65	66.75	38.17	23.81
		(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)
	Const. CV	0.49	0.55	0.44	0.5	0.58	0.45	0.19	0.25	0.15	≤ 0	≤ 0	0.65	≤ 0	1.19	5.98	67.45	39.32	25.32
		(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)
	Rand. CV	0.23	0.15	0.12	0.17	0.08	0.04	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	0.09	1.65	75.78	66.99	61.26
		(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)

Table 14: Results for $L = 1$, $\ell = 1$, `overPenalty = 1`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.08 (4)	0.3 (5)	0.45 (5)	0.04 (4)	0.25 (5)	0.38 (4)	≤ 0 (5)	0.24 (5)	0.38 (5)	≤ 0 (5)	0.25 (5)	1.53 (5)	≤ 0 (4)	1.14 (5)	2.37 (5)	61.28 (5)	25.6 (5)	25 (5)
	Const. CV	0.23 (4)	0.39 (5)	0.52 (5)	0.32 (5)	0.56 (5)	0.71 (5)	0.35 (5)	0.85 (5)	1.11 (5)	0.2 (5)	1.81 (5)	4.51 (5)	0.25 (4)	3.46 (5)	4.82 (5)	61.08 (5)	27.83 (5)	29.59 (5)
	Rand. CV	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	0 (4)	≤ 0 (5)	≤ 0 (5)	0.02 (5)	≤ 0 (5)	≤ 0 (5)	0.04 (5)	1.02 (5)	2.72 (5)	0.76 (5)	4.64 (4)	5.95 (5)	72.23 (4)	58.62 (5)	49.99 (5)
meanBase = 5	Uniform Demands	0.28 (1)	0.39 (1)	0.4 (1)	0.24 (1)	0.39 (1)	0.33 (1)	0.19 (1)	0.36 (1)	0.43 (1)	0.08 (1)	≤ 0 (1)	0.1 (2)	≤ 0 (1)	≤ 0 (2)	2.17 (2)	59.5 (1)	7.99 (2)	8.17 (2)
	Const. CV	0.49 (1)	0.55 (1)	0.51 (1)	0.65 (1)	0.78 (1)	0.74 (1)	0.75 (1)	1.06 (1)	0.98 (1)	0.59 (1)	1.52 (1)	4.36 (2)	0.36 (1)	3.61 (2)	8.09 (2)	60.21 (1)	8.7 (2)	8.45 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	0.06 (1)	0.86 (1)	2.1 (1)	1.25 (1)	7.25 (1)	11.97 (1)	10.62 (2)	59.9 (1)	54.32 (1)
meanBase = 10	Uniform Demands	0.31 (1)	0.44 (1)	0.43 (1)	0.29 (1)	0.41 (1)	0.38 (1)	0.23 (1)	0.42 (1)	0.37 (1)	0.11 (1)	≤ 0 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	2.5 (1)	66.75 (1)	38.17 (1)	23.81 (1)
	Const. CV	0.54 (1)	0.61 (1)	0.5 (1)	0.73 (1)	0.85 (1)	0.76 (1)	0.86 (1)	1.14 (1)	1.16 (1)	0.73 (1)	1.9 (1)	5.07 (1)	0.45 (1)	4.18 (1)	9.95 (1)	67.45 (1)	39.32 (1)	25.32 (1)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.05 (1)	≤ 0 (1)	≤ 0 (1)	0.09 (1)	1.3 (1)	2.98 (1)	1.64 (1)	8.42 (1)	16.04 (1)	75.78 (1)	66.99 (1)	61.26 (1)

Table 15: Results for $L = 1$, $\ell = 1$, overPenalty = 1, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.18	0.42	0.5	0.12	0.36	0.47	≤ 0	0.2	0.36	≤ 0	0.25	0.97	≤ 0	2.15	2.89	77.97	55.12	43.35
		(5)	(5)	(6)	(5)	(5)	(6)	(5)	(5)	(6)	(5)	(6)	(6)	(5)	(6)	(6)	(5)	(6)	(6)
	Const. CV	≤ 0	0.11	0.15	≤ 0	0.04	0.08	≤ 0	≤ 0	0.08	≤ 0	0.12	1.01	≤ 0	1.93	3.12	74.83	49.4	54.31
		(5)	(6)	(6)	(5)	(5)	(6)	(5)	(6)	(6)	(5)	(6)	(6)	(5)	(6)	(6)	(5)	(6)	(6)
	Rand. CV	0.08	0.02	0.03	0.16	≤ 0	≤ 0	0.08	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	0.72	76.66	75.02	73.91
		(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)	(5)
meanBase = 5	Uniform Demands	0.41	0.52	0.45	0.41	0.56	0.54	0.17	0.29	0.31	≤ 0	≤ 0	0.17	≤ 0	3.39	7.33	72.24	60.21	23.95
		(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(3)	(2)	(2)	(3)	(2)	(2)	(3)
	Const. CV	0.15	0.17	0.1	0.1	0.12	0.04	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	0.11	≤ 0	3.9	7.27	68.83	56.59	23.16
		(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(3)	(2)	(2)	(3)	(2)	(2)	(3)
	Rand. CV	0.12	0.24	0.21	0.07	0.23	0.19	≤ 0	0.01	0.07	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	0.08	73.37	69.32	66.49
		(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)
meanBase = 10	Uniform Demands	0.53	0.55	0.47	0.57	0.61	0.52	0.29	0.35	0.25	≤ 0	≤ 0	≤ 0	≤ 0	1.87	11.18	59.8	38.49	14.13
		(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(2)	(1)	(1)	(2)	(1)	(1)	(2)
	Const. CV	0.18	0.18	0.11	0.13	0.13	0.04	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	2.45	11.99	56.04	35.72	14.34
		(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(2)	(1)	(1)	(2)	(1)	(1)	(2)
	Rand. CV	0.21	0.27	0.31	0.19	0.27	0.33	0	0.06	0.09	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	64.51	57.06	51.89
		(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)

Table 16: Results for $L = 1$, $\ell = 1$, `overPenalty = 1`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.3 (6)	0.48 (5)	0.55 (6)	0.39 (5)	0.7 (5)	0.78 (6)	0.41 (5)	0.94 (6)	1.08 (6)	0.65 (5)	4.96 (6)	6.58 (6)	4.34 (5)	9.91 (6)	13.27 (6)	77.97 (5)	55.12 (6)	43.35 (6)
	Const. CV	≤ 0 (5)	0.06 (6)	0.13 (6)	≤ 0 (6)	≤ 0 (6)	0.09 (6)	≤ 0 (5)	≤ 0 (5)	0.22 (6)	0.03 (6)	1.94 (6)	4.11 (6)	1.17 (6)	5.4 (6)	7.42 (6)	74.83 (5)	49.4 (6)	54.31 (6)
	Rand. CV	0.2 (5)	0.1 (6)	0.08 (5)	0.17 (5)	0.14 (6)	0.15 (5)	0.22 (5)	0.13 (5)	0.16 (5)	0.17 (5)	0.47 (6)	1.36 (6)	1.13 (5)	2.38 (5)	4.76 (5)	76.66 (5)	75.02 (5)	73.91 (5)
meanBase = 5	Uniform Demands	0.51 (2)	0.59 (2)	0.55 (2)	0.66 (2)	0.85 (2)	0.84 (2)	0.73 (2)	1.11 (2)	1.2 (2)	1.02 (2)	6.38 (2)	11.72 (2)	5.3 (2)	28.58 (2)	28.05 (2)	72.24 (2)	60.21 (2)	23.95 (3)
	Const. CV	≤ 0 (2)	0.12 (2)	0.07 (2)	≤ 0 (2)	0 (2)	≤ 0 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	0.34 (2)	2.42 (2)	5.26 (2)	2.71 (2)	17.28 (2)	18.95 (2)	68.83 (2)	56.59 (2)	23.16 (3)
	Rand. CV	0.16 (2)	0.26 (2)	0.23 (2)	0.21 (2)	0.35 (2)	0.42 (2)	0.2 (2)	0.35 (2)	0.52 (2)	0.18 (2)	1.03 (2)	2.36 (2)	1.29 (2)	5.49 (2)	8.51 (2)	73.37 (2)	69.32 (2)	66.49 (2)
meanBase = 10	Uniform Demands	0.61 (1)	0.62 (1)	0.54 (1)	0.8 (1)	0.94 (1)	0.86 (1)	0.9 (1)	1.19 (1)	1.24 (1)	1.13 (1)	6.17 (1)	12.37 (1)	6.38 (1)	37.63 (1)	41.7 (2)	59.8 (1)	38.49 (1)	14.13 (2)
	Const. CV	≤ 0 (1)	0.09 (1)	0.06 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	0.25 (1)	2.17 (1)	4.63 (2)	3.01 (1)	21.81 (1)	28.15 (2)	56.04 (1)	35.72 (1)	14.34 (2)
	Rand. CV	0.24 (1)	0.3 (1)	0.31 (1)	0.28 (1)	0.42 (1)	0.5 (1)	0.31 (1)	0.47 (1)	0.52 (1)	0.21 (1)	1.01 (1)	2.29 (1)	1.26 (1)	7.6 (1)	10.62 (1)	64.51 (1)	57.06 (1)	51.89 (1)

Table 17: Results for $L = 1$, $\ell = 1$, overPenalty = 1, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.28 (5)	0.53 (6)	0.52 (5)	0.26 (5)	0.49 (5)	0.59 (6)	0.19 (5)	0.51 (6)	0.57 (6)	≤ 0 (5)	0.08 (6)	0.25 (6)	≤ 0 (5)	0.13 (6)	0.62 (6)	≤ 0 (5)	0.13 (6)	0.62 (6)
	Const. CV	0.37 (5)	0.65 (6)	0.66 (6)	0.34 (5)	0.67 (5)	0.74 (6)	0.23 (5)	0.55 (6)	0.7 (6)	≤ 0 (5)	≤ 0 (6)	0.33 (6)	≤ 0 (5)	0.14 (6)	0.88 (6)	≤ 0 (5)	0.14 (6)	0.88 (6)
	Rand. CV	0.36 (5)	0.36 (5)	0.45 (5)	0.29 (5)	0.3 (5)	0.41 (6)	0.16 (5)	0.17 (5)	0.29 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
meanBase = 5	Uniform Demands	0.49 (2)	0.61 (2)	0.55 (2)	0.52 (2)	0.65 (2)	0.61 (2)	0.42 (2)	0.6 (2)	0.6 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)
	Const. CV	0.59 (2)	0.7 (2)	0.66 (2)	0.63 (2)	0.74 (2)	0.69 (2)	0.48 (2)	0.66 (2)	0.61 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.54 (2)	0.63 (2)	0.66 (2)	0.52 (2)	0.62 (2)	0.66 (2)	0.4 (2)	0.51 (2)	0.54 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.59 (1)	0.61 (1)	0.57 (1)	0.65 (1)	0.68 (1)	0.64 (1)	0.59 (1)	0.65 (1)	0.56 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.7 (1)	0.72 (1)	0.68 (1)	0.74 (1)	0.78 (1)	0.74 (1)	0.65 (1)	0.71 (1)	0.67 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.63 (1)	0.67 (1)	0.69 (1)	0.63 (1)	0.68 (1)	0.71 (1)	0.52 (1)	0.59 (1)	0.58 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 18: Results for $L = 1$, $\ell = 2$, `overPenalty = 0.5`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.09 (5)	0.41 (6)	0.42 (5)	≤ 0 (5)	0.34 (6)	0.35 (6)	≤ 0 (6)	0.1 (6)	0.22 (6)	≤ 0 (5)	0.05 (6)	0.84 (6)	≤ 0 (5)	0.2 (6)	0.88 (6)	≤ 0 (5)	0.13 (6)	0.62 (6)
	Const. CV	0.28 (5)	0.56 (5)	0.63 (6)	0.25 (5)	0.63 (5)	0.71 (6)	0.14 (5)	0.59 (5)	0.71 (6)	≤ 0 (5)	0.14 (6)	0.8 (6)	≤ 0 (6)	≤ 0 (6)	0.55 (6)	≤ 0 (5)	0.14 (6)	0.88 (6)
	Rand. CV	0.39 (5)	0.41 (5)	0.49 (5)	0.4 (5)	0.47 (5)	0.58 (5)	0.46 (5)	0.52 (5)	0.65 (6)	0.41 (5)	0.4 (6)	0.68 (6)	0.3 (5)	0.18 (5)	0.32 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
meanBase = 5	Uniform Demands	0.23 (2)	0.45 (2)	0.44 (2)	0.07 (2)	0.36 (2)	0.4 (2)	≤ 0 (2)	0.12 (2)	0.18 (2)	≤ 0 (2)	≤ 0 (2)	0.12 (2)	≤ 0 (2)	≤ 0 (2)	0.08 (2)	≤ 0 (2)	≤ 0 (2)	0.03 (3)
	Const. CV	0.49 (2)	0.62 (2)	0.58 (2)	0.46 (2)	0.69 (2)	0.66 (2)	0.35 (2)	0.66 (2)	0.67 (2)	0.13 (2)	0 (2)	0.18 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.52 (2)	0.63 (2)	0.67 (2)	0.59 (2)	0.74 (2)	0.8 (2)	0.66 (2)	0.83 (2)	0.92 (2)	0.65 (2)	0.78 (2)	0.91 (2)	0.55 (2)	0.5 (2)	0.48 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.29 (1)	0.47 (1)	0.46 (1)	0.1 (1)	0.36 (1)	0.38 (1)	≤ 0 (1)	0.09 (1)	0.05 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.55 (1)	0.65 (1)	0.62 (1)	0.56 (1)	0.71 (1)	0.68 (1)	0.44 (1)	0.67 (1)	0.65 (1)	0.12 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.59 (1)	0.68 (1)	0.7 (1)	0.67 (1)	0.8 (1)	0.84 (1)	0.76 (1)	0.93 (1)	0.97 (1)	0.77 (1)	0.89 (1)	0.96 (1)	0.66 (1)	0.56 (1)	0.43 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 19: Results for $L = 1$, $\ell = 2$, `overPenalty = 0.5`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.37 (6)	0.62 (6)	0.61 (6)	0.32 (6)	0.62 (6)	0.69 (6)	0.2 (6)	0.59 (6)	0.65 (7)	≤ 0 (6)	≤ 0 (6)	0.34 (7)	≤ 0 (6)	0.1 (6)	0.72 (7)	≤ 0 (6)	0.1 (6)	0.72 (7)
	Const. CV	0.37 (6)	0.6 (6)	0.56 (6)	0.33 (6)	0.6 (6)	0.6 (6)	0.2 (6)	0.54 (6)	0.51 (7)	≤ 0 (6)	≤ 0 (6)	0.16 (7)	≤ 0 (6)	0.07 (6)	0.52 (7)	≤ 0 (6)	0.07 (6)	0.52 (7)
	Rand. CV	0.21 (6)	0.08 (6)	0.09 (7)	0.14 (6)	≤ 0 (6)	≤ 0 (6)	0.04 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	0.55 (2)	0.64 (2)	0.61 (2)	0.57 (2)	0.68 (2)	0.69 (2)	0.46 (2)	0.64 (2)	0.64 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Const. CV	0.57 (2)	0.64 (2)	0.61 (2)	0.57 (2)	0.67 (2)	0.68 (2)	0.47 (2)	0.6 (2)	0.61 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.26 (2)	0.2 (2)	0.16 (2)	0.2 (2)	0.15 (2)	0.08 (2)	0.08 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.64 (1)	0.66 (1)	0.6 (1)	0.69 (1)	0.72 (1)	0.65 (1)	0.63 (1)	0.69 (1)	0.6 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.65 (1)	0.68 (1)	0.63 (1)	0.68 (1)	0.73 (1)	0.68 (1)	0.61 (1)	0.7 (1)	0.63 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.33 (1)	0.21 (1)	0.19 (1)	0.29 (1)	0.16 (1)	0.12 (1)	0.17 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 20: Results for $L = 1$, $\ell = 2$, `overPenalty = 0.5`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.28 (6)	0.54 (6)	0.55 (6)	0.26 (6)	0.56 (6)	0.62 (6)	0.17 (6)	0.54 (6)	0.62 (6)	≤ 0 (6)	0.16 (6)	0.45 (7)	≤ 0 (6)	0.11 (7)	0.53 (7)	≤ 0 (6)	0.1 (6)	0.72 (7)
	Const. CV	0.43 (6)	0.62 (6)	0.61 (7)	0.54 (6)	0.76 (7)	0.77 (6)	0.64 (6)	0.98 (7)	1.01 (7)	0.54 (6)	1.07 (7)	1.87 (7)	0.37 (6)	0.6 (7)	0.95 (7)	≤ 0 (6)	0.07 (6)	0.52 (7)
	Rand. CV	0.03 (6)	≤ 0 (6)	≤ 0 (6)	0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	0.03 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	0.42 (2)	0.56 (2)	0.54 (2)	0.43 (2)	0.63 (2)	0.62 (2)	0.33 (2)	0.6 (2)	0.67 (2)	0.1 (2)	0.03 (2)	0.04 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Const. CV	0.61 (2)	0.66 (2)	0.66 (2)	0.78 (2)	0.88 (2)	0.85 (2)	0.98 (2)	1.17 (2)	1.17 (2)	0.96 (2)	1.23 (2)	1.43 (3)	0.73 (2)	0.69 (2)	0.63 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	0.03 (2)	0.04 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.02 (2)	≤ 0 (2)	≤ 0 (2)	0.07 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.49 (1)	0.58 (1)	0.53 (1)	0.5 (1)	0.65 (1)	0.59 (1)	0.4 (1)	0.65 (1)	0.56 (1)	0.11 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.69 (1)	0.71 (1)	0.66 (1)	0.88 (1)	0.96 (1)	0.9 (1)	1.1 (1)	1.27 (1)	1.22 (1)	1.11 (1)	1.27 (1)	1.38 (1)	0.86 (1)	0.51 (1)	0.49 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.01 (1)	0.04 (1)	0.05 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	0.08 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 21: Results for $L = 1$, $\ell = 2$, `overPenalty = 0.5`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.39 (7)	0.6 (8)	0.59 (8)	0.38 (7)	0.67 (8)	0.6 (8)	0.25 (7)	0.56 (8)	0.57 (8)	≤ 0 (7)	≤ 0 (9)	0.23 (8)	≤ 0 (8)	≤ 0 (9)	0.48 (9)	≤ 0 (8)	≤ 0 (9)	0.48 (9)
	Const. CV	0.07 (7)	0.23 (8)	0.24 (8)	0.03 (7)	0.23 (8)	0.1 (8)	≤ 0 (7)	0.07 (8)	0.06 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	0.06 (9)	0.53 (8)	≤ 0 (8)	0.06 (9)	0.53 (8)
	Rand. CV	0.22 (7)	0.17 (7)	0.26 (8)	0.17 (7)	0.16 (7)	0.26 (7)	0.08 (7)	0.05 (7)	0.15 (7)	≤ 0 (7)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.59 (3)	0.68 (3)	0.61 (3)	0.63 (3)	0.7 (3)	0.67 (3)	0.55 (3)	0.73 (3)	0.63 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.21 (3)	0.23 (3)	0.19 (3)	0.19 (3)	0.2 (3)	0.12 (3)	0.02 (3)	0.04 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.24 (3)	0.32 (3)	0.36 (3)	0.22 (3)	0.33 (3)	0.35 (3)	0.12 (3)	0.21 (3)	0.26 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.64 (2)	0.69 (2)	0.61 (2)	0.69 (2)	0.77 (2)	0.67 (2)	0.64 (2)	0.74 (2)	0.62 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.22 (2)	0.25 (2)	0.18 (2)	0.18 (2)	0.23 (2)	0.13 (2)	0.02 (2)	0.08 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.28 (2)	0.34 (2)	0.36 (2)	0.27 (2)	0.34 (2)	0.39 (2)	0.16 (2)	0.25 (2)	0.3 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 22: Results for $L = 1$, $\ell = 2$, `overPenalty = 0.5`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.47 (7)	0.63 (8)	0.62 (8)	0.6 (7)	0.85 (8)	0.86 (8)	0.75 (7)	1.11 (8)	1.14 (8)	0.65 (7)	1.19 (8)	2 (8)	0.49 (7)	0.71 (8)	1.4 (9)	≤ 0 (8)	≤ 0 (9)	0.48 (9)
	Const. CV	≤ 0 (7)	0.19 (8)	0.23 (8)	≤ 0 (7)	0.12 (8)	0.02 (8)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (7)	≤ 0 (9)	0.19 (8)	≤ 0 (8)	≤ 0 (9)	0.48 (8)	≤ 0 (8)	0.06 (9)	0.53 (8)
	Rand. CV	0.24 (7)	0.21 (7)	0.28 (8)	0.25 (7)	0.28 (7)	0.37 (7)	0.25 (7)	0.3 (7)	0.44 (7)	0.22 (7)	0.1 (7)	0.21 (8)	0.1 (7)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.68 (3)	0.69 (3)	0.65 (3)	0.91 (3)	0.99 (3)	0.9 (3)	1.12 (3)	1.31 (3)	1.26 (3)	1.12 (3)	1.42 (3)	1.69 (3)	0.97 (3)	0.89 (3)	1.05 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.05 (3)	0.19 (3)	0.16 (3)	≤ 0 (3)	0.07 (3)	0.05 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	0.08 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.32 (3)	0.36 (3)	0.37 (3)	0.36 (3)	0.48 (3)	0.52 (3)	0.36 (3)	0.54 (3)	0.66 (3)	0.3 (3)	0.32 (3)	0.28 (3)	0.13 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.71 (2)	0.73 (2)	0.65 (2)	0.93 (2)	0.99 (2)	0.9 (2)	1.15 (2)	1.36 (2)	1.29 (2)	1.21 (2)	1.54 (2)	1.69 (2)	1.07 (2)	1 (2)	1.09 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.05 (2)	0.21 (2)	0.17 (2)	≤ 0 (2)	0.1 (2)	0.07 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.05 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.33 (2)	0.35 (2)	0.38 (2)	0.37 (2)	0.5 (2)	0.52 (2)	0.38 (2)	0.57 (2)	0.66 (2)	0.35 (2)	0.42 (2)	0.45 (2)	0.19 (2)	0.03 (2)	0.05 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 23: Results for $L = 1$, $\ell = 2$, `overPenalty = 0.5`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.23 (4)	0.38 (4)	0.44 (4)	0.19 (4)	0.37 (4)	0.47 (4)	0.1 (4)	0.27 (4)	0.44 (4)	≤ 0 (4)	0.33 (4)	0.79 (4)	≤ 0 (4)	1.28 (5)	2.13 (4)	2.26 (4)	5.56 (5)	7.8 (4)
	Const. CV	0.3 (4)	0.49 (4)	0.58 (4)	0.26 (4)	0.46 (4)	0.62 (4)	0.15 (4)	0.36 (4)	0.57 (4)	≤ 0 (4)	0.11 (4)	0.55 (4)	≤ 0 (4)	1.36 (5)	1.93 (4)	2.21 (4)	5.29 (5)	7.54 (4)
	Rand. CV	0.33 (3)	0.32 (3)	0.37 (3)	0.24 (4)	0.25 (4)	0.31 (3)	0.23 (3)	0.12 (4)	0.22 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	0.07 (4)	1.64 (4)	5.87 (4)	2.79 (4)	3.27 (4)
meanBase = 5	Uniform Demands	0.5 (1)	0.56 (1)	0.51 (1)	0.52 (1)	0.61 (1)	0.56 (1)	0.42 (1)	0.54 (1)	0.5 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	0.63 (2)	4.38 (2)	2.77 (1)	4.46 (2)	5.09 (2)
	Const. CV	0.59 (1)	0.66 (1)	0.59 (1)	0.6 (1)	0.65 (1)	0.62 (1)	0.49 (1)	0.6 (1)	0.52 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	0.58 (2)	3.87 (2)	2.66 (1)	4.28 (2)	4.91 (2)
	Rand. CV	0.49 (1)	0.57 (1)	0.6 (1)	0.46 (1)	0.55 (1)	0.59 (1)	0.34 (1)	0.43 (1)	0.46 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	19.81 (1)	25.38 (1)	25.55 (1)
meanBase = 10	Uniform Demands	0.58 (1)	0.63 (1)	0.58 (1)	0.63 (1)	0.71 (1)	0.65 (1)	0.57 (1)	0.66 (1)	0.61 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.79 (1)	5 (1)	3.66 (1)	5.55 (1)	5.83 (1)
	Const. CV	0.68 (1)	0.73 (1)	0.69 (1)	0.71 (1)	0.8 (1)	0.75 (1)	0.62 (1)	0.73 (1)	0.68 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.64 (1)	4.86 (1)	3.52 (1)	5.21 (1)	5.69 (1)
	Rand. CV	0.6 (1)	0.64 (1)	0.66 (1)	0.58 (1)	0.64 (1)	0.66 (1)	0.47 (1)	0.54 (1)	0.56 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	31.08 (1)	30.86 (1)

Table 24: Results for $L = 1$, $\ell = 2$, `overPenalty = 1`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.09 (3)	0.28 (4)	0.35 (4)	≤ 0 (3)	0.19 (4)	0.28 (4)	≤ 0 (3)	0.05 (4)	0.18 (4)	0.01 (3)	2.63 (4)	4.79 (4)	1.63 (4)	7.29 (5)	11.08 (4)	2.26 (4)	5.56 (5)	7.8 (4)
	Const. CV	0.23 (4)	0.45 (3)	0.54 (4)	0.21 (4)	0.44 (4)	0.62 (4)	0.18 (3)	0.42 (4)	0.65 (4)	0.32 (4)	3.74 (4)	5.35 (4)	4.44 (3)	6.77 (4)	11.49 (4)	2.21 (4)	5.28 (5)	7.54 (4)
	Rand. CV	0.36 (3)	0.37 (4)	0.44 (4)	0.42 (3)	0.42 (4)	0.5 (4)	0.43 (3)	0.47 (3)	0.56 (4)	0.39 (4)	0.76 (4)	1.51 (4)	0.64 (3)	2.42 (4)	5.2 (4)	5.87 (4)	2.79 (4)	3.27 (4)
meanBase = 5	Uniform Demands	0.24 (1)	0.4 (1)	0.41 (1)	0.09 (1)	0.28 (1)	0.34 (1)	0 (1)	0.1 (1)	0.1 (1)	≤ 0 (1)	1.26 (1)	4.6 (2)	1.48 (1)	27.41 (2)	27.16 (2)	2.77 (1)	4.46 (2)	5.09 (2)
	Const. CV	0.46 (1)	0.55 (1)	0.53 (1)	0.46 (1)	0.59 (1)	0.57 (1)	0.37 (1)	0.6 (1)	0.62 (1)	0.33 (1)	3.26 (2)	7.34 (2)	4.84 (1)	28.31 (2)	25.2 (2)	2.66 (1)	4.28 (2)	4.91 (2)
	Rand. CV	0.49 (1)	0.59 (1)	0.61 (1)	0.56 (1)	0.68 (1)	0.71 (1)	0.63 (1)	0.78 (1)	0.82 (1)	0.61 (1)	0.82 (1)	1.25 (1)	0.59 (1)	3.72 (1)	6.57 (1)	19.81 (1)	25.38 (1)	25.55 (1)
meanBase = 10	Uniform Demands	0.3 (1)	0.49 (1)	0.47 (1)	0.12 (1)	0.39 (1)	0.4 (1)	≤ 0 (1)	0.11 (1)	0.12 (1)	≤ 0 (1)	1.44 (1)	4.95 (1)	≤ 0 (1)	30.81 (1)	30.12 (1)	3.66 (1)	5.55 (1)	5.83 (1)
	Const. CV	0.55 (1)	0.66 (1)	0.62 (1)	0.54 (1)	0.73 (1)	0.69 (1)	0.44 (1)	0.68 (1)	0.67 (1)	0.13 (1)	4.24 (1)	8.67 (1)	≤ 0 (1)	33.11 (1)	28.84 (1)	3.52 (1)	5.21 (1)	5.69 (1)
	Rand. CV	0.56 (1)	0.64 (1)	0.66 (1)	0.64 (1)	0.76 (1)	0.78 (1)	0.72 (1)	0.88 (1)	0.91 (1)	0.76 (1)	0.88 (1)	0.93 (1)	0.64 (1)	0.55 (1)	0.44 (1)	≤ 0 (1)	31.08 (1)	30.86 (1)

Table 25: Results for $L = 1$, $\ell = 2$, overPenalty = 1, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.29 (4)	0.47 (5)	0.55 (4)	0.19 (5)	0.46 (4)	0.57 (5)	0.1 (5)	0.37 (5)	0.54 (5)	≤ 0 (5)	0.09 (5)	0.5 (5)	≤ 0 (5)	1.25 (4)	2.1 (5)	58.33 (5)	59.02 (4)	30.67 (5)
	Const. CV	0.29 (4)	0.47 (5)	0.55 (4)	0.19 (5)	0.48 (4)	0.56 (5)	0.09 (5)	0.32 (5)	0.46 (5)	≤ 0 (5)	0.15 (5)	0.59 (5)	≤ 0 (5)	0.86 (5)	1.92 (5)	59.15 (5)	39.81 (5)	31.09 (5)
	Rand. CV	0.13 (5)	≤ 0 (4)	0 (5)	0.06 (5)	≤ 0 (4)	≤ 0 (5)	0.08 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	≤ 0 (5)	1.12 (5)	71.13 (4)	55.51 (5)	39.92 (5)
meanBase = 5	Uniform Demands	0.54 (1)	0.59 (1)	0.57 (1)	0.56 (1)	0.67 (1)	0.6 (1)	0.46 (1)	0.53 (1)	0.52 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	2.55 (2)	53.01 (1)	5.6 (2)	6.02 (2)
	Const. CV	0.55 (1)	0.61 (1)	0.58 (1)	0.55 (1)	0.65 (1)	0.6 (1)	0.44 (1)	0.56 (1)	0.53 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	2.08 (2)	53.86 (1)	5.93 (2)	5.98 (2)
	Rand. CV	0.25 (1)	0.18 (1)	0.14 (1)	0.2 (1)	0.11 (1)	0.08 (1)	0.07 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	7.8 (2)	54.73 (1)	6.58 (2)
meanBase = 10	Uniform Demands	0.6 (1)	0.62 (1)	0.59 (1)	0.64 (1)	0.67 (1)	0.65 (1)	0.54 (1)	0.62 (1)	0.6 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	2.88 (1)	60.27 (1)	26.05 (1)	13.03 (1)
	Const. CV	0.6 (1)	0.66 (1)	0.58 (1)	0.62 (1)	0.7 (1)	0.6 (1)	0.53 (1)	0.64 (1)	0.53 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	2.57 (1)	61.16 (1)	27.22 (1)	14.19 (1)
	Rand. CV	0.29 (1)	0.2 (1)	0.16 (1)	0.26 (1)	0.13 (1)	0.08 (1)	0.13 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	73.06 (1)	61.78 (1)	54.83 (1)

Table 26: Results for $L = 1$, $\ell = 2$, `overPenalty = 1`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.2 (4)	0.4 (5)	0.5 (4)	0.2 (4)	0.4 (4)	0.5 (5)	0.13 (4)	0.4 (5)	0.49 (5)	≤ 0 (5)	0.58 (5)	1.3 (5)	≤ 0 (4)	0.8 (5)	2.02 (5)	58.33 (5)	59.02 (4)	30.67 (5)
	Const. CV	0.33 (4)	0.48 (5)	0.62 (5)	0.44 (4)	0.64 (5)	0.73 (4)	0.55 (5)	0.84 (5)	1.03 (5)	0.59 (4)	3.09 (5)	5.58 (5)	0.41 (5)	3.87 (5)	5.78 (5)	59.15 (5)	39.81 (5)	31.09 (5)
	Rand. CV	0.02 (4)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	0.02 (4)	≤ 0 (5)	≤ 0 (5)	0.15 (4)	0.15 (5)	0.85 (5)	0.33 (5)	3.06 (4)	6.23 (4)	71.13 (4)	55.51 (5)	39.92 (5)
meanBase = 5	Uniform Demands	0.41 (1)	0.5 (1)	0.51 (1)	0.41 (1)	0.54 (1)	0.54 (1)	0.34 (1)	0.52 (1)	0.56 (1)	0.11 (1)	0.01 (2)	0.38 (2)	≤ 0 (1)	≤ 0 (2)	0.38 (2)	53.01 (1)	5.6 (2)	6.02 (2)
	Const. CV	0.59 (1)	0.65 (1)	0.62 (1)	0.74 (1)	0.84 (1)	0.81 (1)	0.92 (1)	1.09 (1)	1.05 (1)	0.87 (1)	2.37 (1)	4.7 (2)	0.63 (1)	3.2 (2)	8.55 (2)	53.86 (1)	5.93 (2)	5.98 (2)
	Rand. CV	0.01 (1)	0.03 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (1)	0.05 (1)	0.16 (1)	0.45 (1)	0.32 (2)	3.43 (1)	6.5 (1)	7.8 (2)	54.73 (1)	6.58 (2)
meanBase = 10	Uniform Demands	0.45 (1)	0.54 (1)	0.53 (1)	0.46 (1)	0.6 (1)	0.57 (1)	0.39 (1)	0.59 (1)	0.58 (1)	0.12 (1)	0.03 (1)	0.46 (1)	≤ 0 (1)	≤ 0 (1)	0.36 (1)	60.27 (1)	26.05 (1)	13.03 (1)
	Const. CV	0.64 (1)	0.69 (1)	0.61 (1)	0.82 (1)	0.91 (1)	0.82 (1)	1.03 (1)	1.2 (1)	1.1 (1)	1.02 (1)	2.72 (1)	5.2 (1)	0.78 (1)	3.84 (1)	9.66 (1)	61.16 (1)	27.22 (1)	14.19 (1)
	Rand. CV	0.02 (1)	0.03 (1)	0.04 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	0.07 (1)	0.11 (1)	0.53 (1)	0.42 (1)	4.29 (1)	8.72 (1)	73.06 (1)	61.78 (1)	54.83 (1)

Table 27: Results for $L = 1$, $\ell = 2$, `overPenalty = 1`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.31 (5)	0.52 (5)	0.6 (6)	0.27 (5)	0.62 (6)	0.59 (6)	0.2 (5)	0.45 (5)	0.57 (6)	≤ 0 (5)	0.01 (6)	0.2 (6)	≤ 0 (5)	0.77 (6)	1.72 (6)	77.07 (5)	64.37 (6)	47.78 (6)
	Const. CV	0.02 (5)	0.17 (5)	0.2 (6)	≤ 0 (5)	0.13 (6)	0.13 (6)	≤ 0 (5)	≤ 0 (5)	0.03 (6)	≤ 0 (5)	≤ 0 (6)	0.37 (6)	≤ 0 (5)	0.79 (6)	2.1 (6)	73.88 (5)	59.88 (6)	43.66 (6)
	Rand. CV	0.19 (5)	0.11 (5)	0.13 (5)	0.11 (5)	0.05 (5)	0.06 (5)	0.02 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	76.16 (5)	74.14 (5)	70.93 (6)
meanBase = 5	Uniform Demands	0.53 (2)	0.63 (2)	0.6 (2)	0.54 (2)	0.68 (2)	0.62 (2)	0.43 (2)	0.62 (2)	0.61 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	0.08 (2)	2.07 (3)	69.74 (2)	54.64 (2)	19.97 (3)
	Const. CV	0.19 (2)	0.23 (2)	0.16 (2)	0.16 (2)	0.18 (2)	0.1 (2)	0.01 (2)	0.03 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	0.02 (2)	2.38 (3)	66.18 (2)	51.19 (2)	19.24 (3)
	Rand. CV	0.19 (2)	0.3 (2)	0.29 (2)	0.15 (2)	0.31 (2)	0.28 (2)	0.05 (2)	0.2 (2)	0.21 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	72.05 (2)	66.94 (2)	63.35 (2)
meanBase = 10	Uniform Demands	0.63 (1)	0.65 (1)	0.59 (1)	0.68 (1)	0.7 (1)	0.66 (1)	0.6 (1)	0.66 (1)	0.58 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	0.81 (2)	54.83 (1)	12.81 (2)	10.41 (2)
	Const. CV	0.22 (1)	0.22 (1)	0.17 (1)	0.18 (1)	0.16 (1)	0.1 (1)	0.04 (1)	0.02 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	0.49 (2)	51.19 (1)	12.48 (2)	10.11 (2)
	Rand. CV	0.26 (1)	0.32 (1)	0.37 (1)	0.24 (1)	0.33 (1)	0.38 (1)	0.13 (1)	0.23 (1)	0.29 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	62.05 (1)	52.62 (1)	46.29 (1)

Table 28: Results for $L = 1$, $\ell = 2$, `overPenalty = 1`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.39 (6)	0.55 (5)	0.63 (6)	0.5 (6)	0.78 (6)	0.85 (6)	0.63 (5)	0.99 (6)	1.1 (6)	0.6 (6)	2.47 (6)	4.33 (6)	2.25 (5)	7.46 (6)	9.92 (6)	77.07 (5)	64.37 (6)	47.78 (6)
	Const. CV	≤ 0 (5)	0.14 (5)	0.19 (6)	≤ 0 (5)	0.05 (6)	0.06 (6)	≤ 0 (6)	≤ 0 (6)	0.1 (6)	0.04 (5)	1.1 (6)	2.31 (6)	0.55 (6)	4.37 (6)	6 (6)	73.88 (5)	59.88 (6)	43.66 (6)
	Rand. CV	0.2 (5)	0.15 (6)	0.19 (6)	0.28 (5)	0.2 (5)	0.22 (5)	0.22 (5)	0.22 (6)	0.27 (5)	0.19 (5)	0.54 (5)	0.73 (6)	0.65 (5)	1.6 (6)	3.12 (5)	76.16 (5)	74.14 (5)	70.93 (6)
meanBase = 5	Uniform Demands	0.59 (2)	0.66 (2)	0.62 (2)	0.77 (2)	0.89 (2)	0.87 (2)	0.95 (2)	1.19 (2)	1.19 (2)	1.09 (2)	2.74 (2)	4.79 (2)	2.53 (2)	16.15 (2)	21.73 (2)	69.74 (2)	54.64 (2)	19.97 (3)
	Const. CV	0.06 (2)	0.18 (2)	0.15 (2)	≤ 0 (2)	0.1 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.01 (2)	0.55 (2)	1.31 (3)	0.9 (2)	7.47 (2)	12.94 (2)	66.18 (2)	51.19 (2)	19.24 (3)
	Rand. CV	0.25 (2)	0.3 (2)	0.32 (2)	0.28 (2)	0.43 (2)	0.41 (2)	0.27 (2)	0.43 (2)	0.56 (2)	0.25 (2)	0.53 (2)	1.01 (2)	0.49 (2)	3.14 (2)	5.27 (2)	72.05 (2)	66.94 (2)	63.35 (2)
meanBase = 10	Uniform Demands	0.69 (1)	0.69 (1)	0.64 (1)	0.88 (1)	0.95 (1)	0.87 (1)	1.11 (1)	1.27 (1)	1.22 (1)	1.17 (1)	2.23 (1)	3.63 (1)	2.44 (1)	18.59 (1)	28.42 (2)	54.83 (1)	12.81 (2)	10.41 (2)
	Const. CV	0.06 (1)	0.17 (1)	0.15 (1)	≤ 0 (1)	0.07 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.29 (1)	0.7 (2)	0.94 (1)	6.11 (1)	13.61 (2)	51.19 (1)	12.48 (2)	10.11 (2)
	Rand. CV	0.3 (1)	0.34 (1)	0.36 (1)	0.35 (1)	0.46 (1)	0.5 (1)	0.35 (1)	0.53 (1)	0.61 (1)	0.33 (1)	0.54 (1)	0.91 (1)	0.47 (1)	3.72 (1)	6.13 (1)	62.05 (1)	52.62 (1)	46.29 (1)

Table 29: Results for $L = 1$, $\ell = 2$, `overPenalty = 1`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.19 (5)	0.46 (5)	0.38 (6)	0.16 (5)	0.55 (5)	0.42 (6)	0.01 (5)	0.37 (5)	0.25 (6)	≤ 0 (5)	0.06 (6)	0.6 (6)	≤ 0 (5)	0.06 (6)	0.6 (6)	≤ 0 (5)	0.06 (6)	0.6 (6)
	Const. CV	0.26 (5)	0.55 (5)	0.52 (6)	0.22 (5)	0.6 (5)	0.56 (6)	≤ 0 (5)	0.21 (6)	0.32 (6)	≤ 0 (5)	0.11 (6)	0.65 (6)	≤ 0 (5)	0.11 (6)	0.65 (6)	≤ 0 (5)	0.11 (6)	0.65 (6)
	Rand. CV	0.24 (5)	0.26 (5)	0.32 (6)	0.17 (5)	0.17 (5)	0.3 (5)	0.05 (5)	≤ 0 (5)	0.07 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
meanBase = 5	Uniform Demands	0.36 (2)	0.47 (2)	0.39 (2)	0.37 (2)	0.55 (2)	0.44 (2)	0.18 (2)	0.32 (2)	0.23 (2)	≤ 0 (2)	≤ 0 (2)	0.2 (2)	≤ 0 (2)	≤ 0 (2)	0.2 (2)	≤ 0 (2)	≤ 0 (2)	0.2 (2)
	Const. CV	0.45 (2)	0.59 (2)	0.48 (2)	0.46 (2)	0.59 (2)	0.51 (2)	0.16 (2)	0.27 (2)	0.14 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.38 (2)	0.45 (2)	0.45 (2)	0.35 (2)	0.44 (2)	0.43 (2)	0.07 (2)	0.09 (2)	0.08 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.42 (1)	0.47 (1)	0.36 (1)	0.43 (1)	0.52 (1)	0.37 (1)	0.18 (1)	0.19 (1)	0.1 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.51 (1)	0.52 (1)	0.45 (1)	0.52 (1)	0.55 (1)	0.46 (1)	0.15 (1)	0.15 (1)	0.09 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.45 (1)	0.49 (1)	0.5 (1)	0.41 (1)	0.45 (1)	0.46 (1)	0.06 (1)	0.03 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 30: Results for $L = 3$, $\ell = 1$, `overPenalty = 0.5`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		`chiBase` = 0			`chiBase` = 1			`chiBase` = 2			`chiBase` = 5			`chiBase` = 7			`chiBase` = 999		
	`CVBase` →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
`meanBase` = 1	Uniform Demands	≤ 0 (5)	0.31 (6)	0.29 (6)	≤ 0 (5)	0.13 (6)	0.17 (6)	≤ 0 (5)	0.12 (5)	0.15 (6)	≤ 0 (5)	0.08 (6)	0.46 (6)	≤ 0 (5)	≤ 0 (6)	0.78 (6)	≤ 0 (5)	0.06 (6)	0.6 (6)
	Const. CV	0.15 (5)	0.49 (5)	0.49 (6)	0.07 (5)	0.45 (5)	0.45 (6)	0.02 (5)	0.37 (6)	0.64 (6)	≤ 0 (6)	0.03 (6)	0.81 (6)	≤ 0 (5)	≤ 0 (6)	0.48 (6)	≤ 0 (5)	0.11 (6)	0.65 (6)
	Rand. CV	0.27 (5)	0.32 (5)	0.39 (6)	0.32 (5)	0.34 (5)	0.45 (6)	0.38 (5)	0.39 (5)	0.52 (6)	0.31 (5)	0.15 (5)	0.32 (5)	0.14 (5)	≤ 0 (6)	0.23 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
`meanBase` = 5	Uniform Demands	0.09 (2)	0.35 (2)	0.3 (2)	≤ 0 (2)	0.1 (2)	0.07 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.14 (2)	≤ 0 (2)	≤ 0 (2)	0.12 (2)	≤ 0 (2)	≤ 0 (2)	0.2 (2)
	Const. CV	0.29 (2)	0.47 (2)	0.43 (2)	0.26 (2)	0.43 (2)	0.36 (2)	0.21 (2)	0.42 (2)	0.5 (2)	0.05 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.38 (2)	0.5 (2)	0.51 (2)	0.45 (2)	0.57 (2)	0.6 (2)	0.5 (2)	0.65 (2)	0.72 (2)	0.44 (2)	0.38 (2)	0.4 (2)	0.3 (2)	0.19 (2)	0.2 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
`meanBase` = 10	Uniform Demands	0.07 (1)	0.33 (1)	0.28 (1)	≤ 0 (1)	0.01 (1)	0.02 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.31 (1)	0.46 (1)	0.41 (1)	0.25 (1)	0.39 (1)	0.34 (1)	0.2 (1)	0.26 (1)	0.34 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.4 (1)	0.5 (1)	0.53 (1)	0.46 (1)	0.56 (1)	0.59 (1)	0.53 (1)	0.64 (1)	0.69 (1)	0.48 (1)	0.41 (1)	0.39 (1)	0.33 (1)	0.18 (1)	0.07 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 31: Results for $L = 3$, $\ell = 1$, `overPenalty = 0.5`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.25 (6)	0.53 (7)	0.46 (6)	0.21 (6)	0.53 (6)	0.57 (7)	0.01 (6)	0.34 (6)	0.36 (6)	≤ 0 (6)	0.31 (7)	0.51 (7)	≤ 0 (6)	0.31 (7)	0.51 (7)	≤ 0 (6)	0.31 (7)	0.51 (7)
	Const. CV	0.27 (6)	0.51 (7)	0.48 (6)	0.21 (6)	0.55 (6)	0.43 (7)	≤ 0 (6)	0.33 (6)	0.32 (6)	≤ 0 (6)	0.22 (7)	0.45 (7)	≤ 0 (6)	0.22 (7)	0.45 (7)	≤ 0 (6)	0.22 (7)	0.45 (7)
	Rand. CV	0.14 (6)	0.02 (6)	0.04 (6)	0.08 (6)	≤ 0 (6)	≤ 0 (6)	0.01 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	0.03 (6)	≤ 0 (6)	≤ 0 (6)	0.03 (6)	≤ 0 (6)	≤ 0 (6)	0.03 (6)
meanBase = 5	Uniform Demands	0.43 (2)	0.52 (2)	0.48 (2)	0.43 (2)	0.57 (2)	0.51 (2)	0.2 (2)	0.32 (2)	0.3 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Const. CV	0.44 (2)	0.56 (2)	0.48 (2)	0.41 (2)	0.59 (2)	0.48 (2)	0.14 (2)	0.32 (2)	0.17 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.16 (2)	0.14 (2)	0.11 (2)	0.09 (2)	0.09 (2)	0.02 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.47 (1)	0.48 (1)	0.44 (1)	0.48 (1)	0.51 (1)	0.46 (1)	0.2 (1)	0.24 (1)	0.16 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.48 (1)	0.53 (1)	0.47 (1)	0.48 (1)	0.54 (1)	0.48 (1)	0.14 (1)	0.18 (1)	0.18 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.21 (1)	0.12 (1)	0.09 (1)	0.15 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 32: Results for $L = 3$, $\ell = 1$, `overPenalty = 0.5`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.14 (6)	0.45 (7)	0.41 (6)	0.08 (6)	0.43 (7)	0.37 (6)	0.05 (6)	0.37 (6)	0.47 (7)	≤ 0 (7)	0.11 (7)	0.45 (6)	≤ 0 (6)	0.17 (7)	0.39 (7)	≤ 0 (6)	0.31 (7)	0.51 (7)
	Const. CV	0.34 (6)	0.62 (6)	0.52 (7)	0.45 (6)	0.74 (7)	0.74 (7)	0.47 (6)	0.95 (6)	1.16 (7)	0.28 (6)	0.67 (6)	1.06 (7)	0.01 (6)	0.38 (7)	0.53 (7)	≤ 0 (6)	0.22 (7)	0.45 (7)
	Rand. CV	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	0.04 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	0.03 (6)
meanBase = 5	Uniform Demands	0.27 (2)	0.43 (2)	0.43 (2)	0.22 (2)	0.41 (2)	0.37 (2)	0.21 (2)	0.31 (2)	0.48 (2)	0.04 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Const. CV	0.52 (2)	0.61 (2)	0.54 (2)	0.69 (2)	0.84 (2)	0.79 (2)	0.79 (2)	1.04 (2)	1.09 (2)	0.64 (2)	0.62 (2)	0.64 (3)	0.35 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.07 (2)	≤ 0 (2)	≤ 0 (2)	0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.29 (1)	0.39 (1)	0.39 (1)	0.24 (1)	0.39 (1)	0.33 (1)	0.18 (1)	0.31 (1)	0.33 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.55 (1)	0.59 (1)	0.52 (1)	0.72 (1)	0.84 (1)	0.78 (1)	0.83 (1)	1.11 (1)	1.16 (1)	0.7 (1)	0.6 (1)	0.61 (2)	0.38 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.02 (1)	≤ 0 (1)	≤ 0 (1)	0.09 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 33: Results for $L = 3$, $\ell = 1$, `overPenalty = 0.5`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.34 (8)	0.56 (8)	0.56 (8)	0.27 (7)	0.56 (8)	0.57 (8)	0.07 (7)	0.36 (8)	0.39 (8)	≤ 0 (7)	≤ 0 (9)	0.59 (8)	≤ 0 (7)	≤ 0 (9)	0.59 (8)	≤ 0 (7)	≤ 0 (9)	0.59 (8)
	Const. CV	0.09 (8)	0.19 (8)	0.22 (8)	≤ 0 (7)	0.19 (9)	0.17 (8)	≤ 0 (7)	≤ 0 (8)	0.15 (8)	≤ 0 (8)	0.03 (9)	0.44 (8)	≤ 0 (8)	0.03 (9)	0.44 (8)	≤ 0 (8)	0.03 (9)	0.44 (8)
	Rand. CV	0.19 (7)	0.12 (8)	0.16 (7)	0.13 (7)	0.1 (7)	0.21 (8)	0.12 (8)	≤ 0 (7)	≤ 0 (7)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.54 (3)	0.57 (3)	0.49 (3)	0.55 (3)	0.63 (3)	0.56 (3)	0.33 (3)	0.43 (3)	0.31 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.15 (3)	0.22 (3)	0.12 (3)	0.1 (3)	0.2 (3)	0.08 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.21 (3)	0.25 (3)	0.28 (3)	0.19 (3)	0.25 (3)	0.3 (3)	0.02 (3)	0.08 (3)	0.16 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.51 (2)	0.58 (2)	0.48 (2)	0.54 (2)	0.66 (2)	0.53 (2)	0.27 (2)	0.41 (2)	0.28 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.17 (2)	0.18 (2)	0.09 (2)	0.11 (2)	0.14 (2)	0.03 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.2 (2)	0.29 (2)	0.29 (2)	0.16 (2)	0.3 (2)	0.32 (2)	0.02 (2)	0.15 (2)	0.2 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 34: Results for $L = 3$, $\ell = 1$, `overPenalty = 0.5`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.41 (7)	0.61 (8)	0.6 (8)	0.52 (7)	0.86 (8)	0.89 (8)	0.56 (8)	1.12 (8)	1.3 (8)	0.42 (8)	0.74 (9)	1.2 (8)	0.22 (8)	0.36 (8)	0.71 (8)	≤ 0 (7)	≤ 0 (9)	0.59 (8)
	Const. CV	≤ 0 (7)	0.13 (8)	0.2 (8)	≤ 0 (7)	0.06 (8)	0.16 (8)	≤ 0 (7)	0.01 (8)	0.07 (8)	≤ 0 (8)	0.04 (8)	0.2 (8)	≤ 0 (8)	≤ 0 (9)	0.39 (8)	≤ 0 (8)	0.03 (9)	0.44 (8)
	Rand. CV	0.17 (7)	0.2 (7)	0.17 (7)	0.23 (7)	0.24 (7)	0.34 (8)	0.19 (7)	0.22 (7)	0.36 (8)	0.13 (8)	≤ 0 (8)	0.12 (8)	0.04 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (7)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.6 (3)	0.65 (3)	0.59 (3)	0.79 (3)	0.96 (3)	0.89 (3)	0.88 (3)	1.28 (3)	1.26 (3)	0.72 (3)	0.92 (3)	1.07 (3)	0.49 (3)	0.48 (3)	0.46 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	≤ 0 (3)	0.15 (3)	0.09 (3)	≤ 0 (3)	0.07 (3)	0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.22 (3)	0.35 (3)	0.32 (3)	0.26 (3)	0.49 (3)	0.52 (3)	0.27 (3)	0.51 (3)	0.56 (3)	0.13 (3)	0.23 (3)	0.15 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.62 (2)	0.66 (2)	0.55 (2)	0.82 (2)	0.99 (2)	0.89 (2)	0.89 (2)	1.27 (2)	1.29 (2)	0.84 (2)	0.76 (2)	0.91 (2)	0.66 (2)	0.19 (2)	0.25 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (2)	0.11 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.04 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.24 (2)	0.35 (2)	0.34 (2)	0.29 (2)	0.5 (2)	0.55 (2)	0.31 (2)	0.55 (2)	0.6 (2)	0.18 (2)	0.15 (2)	0.05 (2)	0.02 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 35: Results for $L = 3$, $\ell = 1$, `overPenalty = 0.5`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.11 (4)	0.37 (4)	0.32 (4)	0.08 (4)	0.32 (4)	0.38 (4)	≤ 0 (4)	0.19 (4)	0.33 (4)	≤ 0 (4)	0.62 (5)	1.78 (4)	≤ 0 (4)	1.63 (5)	3.54 (4)	2.42 (4)	5.75 (5)	8.18 (4)
	Const. CV	0.18 (4)	0.43 (4)	0.44 (4)	0.13 (4)	0.38 (4)	0.45 (4)	≤ 0 (4)	0.18 (4)	0.36 (4)	≤ 0 (4)	0.55 (5)	1.57 (4)	≤ 0 (4)	1.49 (5)	3.28 (4)	2.34 (4)	5.74 (5)	7.92 (4)
	Rand. CV	0.17 (4)	0.17 (4)	0.19 (4)	0.2 (3)	0.09 (4)	0.13 (4)	0.04 (3)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	0.26 (4)	≤ 0 (4)	0.88 (4)	4.12 (4)	8.87 (4)	7.96 (4)	7.32 (4)
meanBase = 5	Uniform Demands	0.3 (1)	0.35 (1)	0.29 (1)	0.27 (1)	0.34 (1)	0.31 (1)	0.05 (1)	0.09 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (2)	1.46 (2)	≤ 0 (1)	4.97 (2)	6.27 (2)	3.23 (1)	5.5 (2)	6.3 (2)
	Const. CV	0.38 (1)	0.45 (1)	0.4 (1)	0.34 (1)	0.43 (1)	0.42 (1)	0.04 (1)	0.06 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	1.24 (2)	≤ 0 (1)	4.76 (2)	5.69 (2)	3.07 (1)	5.25 (2)	5.7 (2)
	Rand. CV	0.3 (1)	0.36 (1)	0.37 (1)	0.24 (1)	0.28 (1)	0.3 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.31 (1)	6.65 (1)	21.3 (1)	26.87 (1)	27.33 (1)
meanBase = 10	Uniform Demands	0.33 (1)	0.36 (1)	0.29 (1)	0.3 (1)	0.35 (1)	0.27 (1)	0.04 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	1.84 (1)	≤ 0 (1)	6.25 (1)	7.64 (1)	4.48 (1)	6.9 (1)	7.68 (1)
	Const. CV	0.4 (1)	0.44 (1)	0.37 (1)	0.36 (1)	0.42 (1)	0.35 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	1.47 (1)	≤ 0 (1)	5.8 (1)	7.05 (1)	4.28 (1)	6.46 (1)	7.1 (1)
	Rand. CV	0.38 (1)	0.42 (1)	0.43 (1)	0.33 (1)	0.38 (1)	0.38 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.62 (1)	7.76 (1)	≤ 0 (1)	33.12 (1)	32.93 (1)

Table 36: Results for $L = 3$, $\ell = 1$, `overPenalty = 1`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.03 (3)	0.26 (4)	0.25 (4)	≤ 0 (3)	0.14 (4)	0.19 (4)	≤ 0 (4)	≤ 0 (4)	0.24 (4)	0.14 (4)	5.37 (4)	6.58 (4)	4.8 (4)	8.74 (4)	16.59 (4)	2.42 (4)	5.75 (5)	8.18 (4)
	Const. CV	0.11 (4)	0.32 (4)	0.43 (4)	0.13 (3)	0.27 (4)	0.41 (4)	0.08 (4)	0.35 (4)	0.56 (4)	1.14 (4)	5.75 (4)	7.3 (4)	4.92 (4)	9.49 (4)	14.61 (4)	2.34 (4)	5.74 (5)	7.92 (4)
	Rand. CV	0.25 (3)	0.22 (4)	0.26 (4)	0.25 (3)	0.24 (4)	0.29 (4)	0.28 (3)	0.27 (3)	0.37 (4)	0.25 (3)	0.68 (4)	2.03 (4)	0.33 (4)	4.46 (4)	6.46 (4)	8.87 (4)	7.96 (4)	7.32 (4)
meanBase = 5	Uniform Demands	0.03 (1)	0.23 (1)	0.24 (1)	≤ 0 (1)	≤ 0 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.06 (1)	10.19 (1)	18.19 (2)	7.55 (1)	32.07 (1)	26.65 (2)	3.23 (1)	5.5 (2)	6.3 (2)
	Const. CV	0.24 (1)	0.36 (1)	0.37 (1)	0.23 (1)	0.34 (1)	0.33 (1)	0.17 (1)	0.33 (1)	0.38 (1)	1.25 (1)	13.29 (1)	19.36 (2)	12.5 (1)	27.7 (2)	19.51 (2)	3.07 (1)	5.25 (2)	5.7 (2)
	Rand. CV	0.31 (1)	0.39 (1)	0.42 (1)	0.38 (1)	0.44 (1)	0.46 (1)	0.42 (1)	0.51 (1)	0.55 (1)	0.38 (1)	1.21 (1)	2.6 (1)	0.65 (1)	7.38 (1)	14.4 (1)	21.3 (1)	26.87 (1)	27.33 (1)
meanBase = 10	Uniform Demands	0.03 (1)	0.23 (1)	0.21 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	11.18 (1)	20.54 (1)	≤ 0 (1)	36.68 (1)	30.65 (1)	4.48 (1)	6.9 (1)	7.68 (1)
	Const. CV	0.24 (1)	0.37 (1)	0.34 (1)	0.19 (1)	0.3 (1)	0.24 (1)	0.14 (1)	0.18 (1)	0.46 (1)	≤ 0 (1)	15.8 (1)	22.77 (1)	≤ 0 (1)	32.59 (1)	22.6 (1)	4.28 (1)	6.46 (1)	7.1 (1)
	Rand. CV	0.35 (1)	0.44 (1)	0.46 (1)	0.4 (1)	0.49 (1)	0.53 (1)	0.46 (1)	0.55 (1)	0.6 (1)	0.41 (1)	0.28 (1)	0.21 (1)	0.28 (1)	0.09 (1)	16.19 (1)	≤ 0 (1)	33.12 (1)	32.93 (1)

Table 37: Results for $L = 3$, $\ell = 1$, `overPenalty = 1`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.3 (4)	0.35 (5)	0.55 (5)	0.16 (5)	0.28 (5)	0.47 (5)	0.01 (5)	0.11 (5)	0.22 (5)	≤ 0 (5)	0.65 (5)	1.58 (5)	≤ 0 (5)	1.78 (5)	2.71 (5)	61.4 (5)	43.4 (5)	42.84 (5)
	Const. CV	0.28 (4)	0.47 (5)	0.6 (5)	0.14 (5)	0.37 (5)	0.49 (5)	≤ 0 (5)	0.15 (5)	0.22 (5)	≤ 0 (5)	0.57 (5)	1.45 (5)	≤ 0 (5)	1.58 (5)	2.56 (5)	60.8 (5)	47.01 (5)	42.82 (5)
	Rand. CV	0.18 (4)	0.05 (4)	0.02 (5)	0.15 (4)	≤ 0 (5)	≤ 0 (4)	0.02 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	≤ 0 (5)	0.14 (5)	≤ 0 (4)	0.21 (5)	1.91 (5)	72.12 (4)	59.92 (5)	53.66 (5)
meanBase = 5	Uniform Demands	0.35 (1)	0.41 (1)	0.39 (1)	0.31 (1)	0.39 (1)	0.34 (1)	0.05 (1)	0.08 (1)	0.02 (1)	≤ 0 (1)	≤ 0 (2)	1.08 (2)	≤ 0 (1)	1.56 (2)	5.42 (2)	59.13 (1)	12.62 (2)	11.23 (2)
	Const. CV	0.36 (1)	0.43 (1)	0.4 (1)	0.3 (1)	0.38 (1)	0.36 (1)	0.03 (1)	0.03 (1)	0.06 (2)	≤ 0 (1)	≤ 0 (2)	0.55 (2)	≤ 0 (1)	0.99 (2)	4.29 (2)	59.83 (1)	14.85 (2)	12.34 (2)
	Rand. CV	0.1 (1)	0.03 (1)	0.02 (1)	0.02 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	0.11 (1)	1.66 (1)	35.04 (2)	59.47 (1)	53.77 (1)
meanBase = 10	Uniform Demands	0.39 (1)	0.44 (1)	0.39 (1)	0.37 (1)	0.42 (1)	0.37 (1)	0.08 (1)	0.16 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	1.32 (1)	≤ 0 (1)	1.8 (1)	6.73 (1)	66.43 (1)	37.84 (1)	24.41 (1)
	Const. CV	0.4 (1)	0.5 (1)	0.44 (1)	0.36 (1)	0.49 (1)	0.44 (1)	0.04 (1)	0.19 (1)	0.08 (1)	≤ 0 (1)	≤ 0 (1)	0.97 (1)	≤ 0 (1)	1.44 (1)	5.68 (1)	67.11 (1)	39.09 (1)	25.99 (1)
	Rand. CV	0.14 (1)	0.06 (1)	0.01 (1)	0.05 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.16 (1)	1.94 (1)	75.59 (1)	66.59 (1)	60.84 (1)

Table 38: Results for $L = 3$, $\ell = 1$, overPenalty = 1, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.16 (4)	0.29 (5)	0.49 (5)	0.12 (4)	0.28 (5)	0.37 (4)	0.06 (5)	0.25 (5)	0.44 (5)	≤ 0 (5)	0.23 (5)	1.28 (5)	≤ 0 (4)	1.21 (5)	2.33 (5)	61.4 (5)	43.4 (5)	42.84 (5)
	Const. CV	0.31 (4)	0.49 (5)	0.6 (5)	0.36 (5)	0.66 (5)	0.77 (5)	0.36 (5)	1 (5)	1.23 (5)	0.21 (5)	2 (5)	4.2 (5)	0.29 (4)	3.36 (5)	4.45 (5)	60.8 (5)	47.01 (5)	42.82 (5)
	Rand. CV	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	0.04 (4)	≤ 0 (5)	≤ 0 (5)	0.02 (5)	≤ 0 (5)	≤ 0 (5)	0.05 (5)	1.12 (5)	2.69 (5)	0.81 (5)	4.52 (4)	5.68 (5)	72.12 (4)	59.92 (5)	53.66 (5)
meanBase = 5	Uniform Demands	0.21 (1)	0.32 (1)	0.33 (1)	0.17 (1)	0.29 (1)	0.27 (1)	0.13 (1)	0.23 (1)	0.31 (2)	≤ 0 (1)	≤ 0 (2)	0.37 (2)	≤ 0 (1)	0.05 (2)	2.78 (2)	59.13 (1)	12.62 (2)	11.23 (2)
	Const. CV	0.42 (1)	0.48 (1)	0.46 (1)	0.57 (1)	0.67 (1)	0.67 (2)	0.67 (1)	0.91 (1)	0.96 (2)	0.51 (1)	1.93 (2)	5.12 (2)	0.32 (1)	4.01 (2)	8.54 (2)	59.83 (1)	14.85 (2)	12.34 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (2)	1.29 (1)	2.4 (1)	0.97 (2)	6.79 (1)	11.7 (1)	35.04 (2)	59.47 (1)	53.77 (1)
meanBase = 10	Uniform Demands	0.24 (1)	0.35 (1)	0.34 (1)	0.2 (1)	0.33 (1)	0.25 (1)	0.15 (1)	0.25 (1)	0.29 (1)	0 (1)	≤ 0 (1)	0.32 (1)	≤ 0 (1)	0.16 (1)	3.58 (1)	66.43 (1)	37.84 (1)	24.41 (1)
	Const. CV	0.46 (1)	0.56 (1)	0.5 (1)	0.64 (1)	0.78 (1)	0.72 (1)	0.76 (1)	1.03 (1)	1.05 (1)	0.62 (1)	2.32 (1)	6.02 (1)	0.41 (1)	4.65 (1)	10.68 (1)	67.11 (1)	39.09 (1)	25.99 (1)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	0.1 (1)	1.38 (1)	3.31 (1)	1.6 (1)	8.71 (1)	15.54 (1)	75.59 (1)	66.59 (1)	60.84 (1)

Table 39: Results for $L = 3$, $\ell = 1$, `overPenalty = 1`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.22 (5)	0.45 (6)	0.49 (6)	0.18 (5)	0.53 (5)	0.44 (6)	0.01 (5)	0.33 (5)	0.32 (6)	≤ 0 (5)	0.23 (6)	0.94 (6)	≤ 0 (5)	1.97 (6)	3.02 (6)	77.99 (5)	57.99 (6)	59.61 (6)
	Const. CV	≤ 0 (5)	0.18 (6)	0.21 (6)	≤ 0 (5)	0.06 (5)	0.05 (6)	≤ 0 (5)	≤ 0 (5)	0.11 (6)	≤ 0 (5)	0.31 (6)	0.98 (6)	≤ 0 (5)	1.83 (6)	3.01 (6)	74.84 (5)	53.77 (6)	55.1 (6)
	Rand. CV	0.09 (5)	0.03 (5)	0.11 (5)	0.12 (5)	≤ 0 (5)	0.06 (5)	0.03 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	0.84 (5)	76.67 (5)	75.03 (5)	73.86 (5)
meanBase = 5	Uniform Demands	0.39 (2)	0.5 (2)	0.44 (2)	0.39 (2)	0.54 (2)	0.5 (2)	0.16 (2)	0.29 (2)	0.29 (2)	≤ 0 (2)	≤ 0 (2)	0.5 (2)	≤ 0 (2)	3.4 (2)	8.85 (2)	72.12 (2)	59.8 (2)	51.78 (2)
	Const. CV	0.09 (2)	0.14 (2)	0.06 (2)	0 (2)	0.08 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.43 (2)	≤ 0 (2)	4.07 (2)	9.13 (2)	68.67 (2)	56.16 (2)	48.26 (2)
	Rand. CV	0.12 (2)	0.21 (2)	0.26 (2)	0.07 (2)	0.2 (2)	0.26 (2)	≤ 0 (2)	≤ 0 (2)	0.14 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.38 (2)	73.27 (2)	69.21 (2)	66.23 (2)
meanBase = 10	Uniform Demands	0.44 (1)	0.51 (1)	0.39 (1)	0.44 (1)	0.53 (1)	0.41 (1)	0.16 (1)	0.2 (1)	0.12 (1)	≤ 0 (1)	≤ 0 (1)	0.12 (2)	≤ 0 (1)	2.83 (1)	9.68 (2)	59.44 (1)	38.01 (1)	17.11 (2)
	Const. CV	0.12 (1)	0.15 (1)	0.03 (1)	0.06 (1)	0.08 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.07 (2)	≤ 0 (1)	3.61 (1)	9.63 (2)	55.74 (1)	35.25 (1)	16.97 (2)
	Rand. CV	0.16 (1)	0.22 (1)	0.23 (1)	0.13 (1)	0.19 (1)	0.21 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.05 (1)	64.29 (1)	56.64 (1)	51.45 (1)

Table 40: Results for $L = 3$, $\ell = 1$, `overPenalty = 1`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.33 (6)	0.51 (6)	0.54 (6)	0.44 (5)	0.69 (6)	0.77 (6)	0.46 (5)	0.93 (6)	1.18 (6)	0.83 (5)	4.83 (6)	6.18 (6)	2.37 (6)	9.51 (6)	13.59 (6)	77.99 (5)	57.99 (6)	59.61 (6)
	Const. CV	≤ 0 (5)	0.13 (6)	0.18 (6)	≤ 0 (6)	0.06 (6)	0.17 (6)	≤ 0 (5)	0.06 (6)	0.04 (6)	0.13 (6)	2.16 (6)	3.88 (6)	1.32 (6)	5.42 (6)	7.73 (6)	74.84 (5)	53.77 (6)	55.1 (6)
	Rand. CV	0.17 (5)	0.1 (6)	0.15 (5)	0.15 (5)	0.12 (6)	0.26 (5)	0.18 (5)	0.11 (5)	0.29 (5)	0.16 (5)	0.5 (6)	1.41 (6)	1.15 (5)	2.52 (5)	4.69 (5)	76.67 (5)	75.03 (5)	73.86 (5)
meanBase = 5	Uniform Demands	0.52 (2)	0.58 (2)	0.5 (2)	0.69 (2)	0.84 (2)	0.78 (2)	0.76 (2)	1.07 (2)	1.18 (2)	1.28 (2)	6.63 (2)	11.78 (2)	6.41 (2)	26.08 (2)	26.96 (2)	72.12 (2)	59.8 (2)	51.78 (2)
	Const. CV	≤ 0 (2)	0.07 (2)	0.07 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.02 (2)	≤ 0 (2)	≤ 0 (2)	0.49 (2)	2.94 (2)	5.61 (2)	2.7 (2)	15.45 (2)	17.4 (2)	68.67 (2)	56.16 (2)	48.26 (2)
	Rand. CV	0.18 (2)	0.26 (2)	0.3 (2)	0.21 (2)	0.33 (2)	0.42 (2)	0.21 (2)	0.35 (2)	0.48 (2)	0.16 (2)	1.34 (2)	2.48 (2)	1.59 (2)	5.23 (2)	8.32 (2)	73.27 (2)	69.21 (2)	66.23 (2)
meanBase = 10	Uniform Demands	0.55 (1)	0.58 (1)	0.46 (1)	0.73 (1)	0.85 (1)	0.76 (1)	0.83 (1)	1.14 (1)	1.12 (1)	1.19 (1)	7.15 (1)	12.98 (1)	7.36 (1)	34.54 (1)	36.71 (2)	59.44 (1)	38.01 (1)	17.11 (2)
	Const. CV	≤ 0 (1)	0.09 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (1)	0.31 (1)	2.48 (1)	5.32 (2)	3.3 (1)	20.34 (1)	23.54 (2)	55.74 (1)	35.25 (1)	16.97 (2)
	Rand. CV	0.2 (1)	0.26 (1)	0.25 (1)	0.23 (1)	0.36 (1)	0.4 (1)	0.24 (1)	0.4 (1)	0.44 (1)	0.19 (1)	1.25 (1)	2.44 (1)	1.5 (1)	7.22 (1)	10.55 (1)	64.29 (1)	56.64 (1)	51.45 (1)

Table 41: Results for $L = 3$, $\ell = 1$, `overPenalty = 1`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.3 (5)	0.52 (6)	0.49 (6)	0.29 (5)	0.62 (5)	0.56 (6)	0.19 (5)	0.49 (6)	0.53 (6)	≤ 0 (5)	0.2 (6)	0.32 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)
	Const. CV	0.39 (5)	0.6 (6)	0.65 (6)	0.36 (5)	0.72 (5)	0.71 (6)	0.23 (5)	0.53 (6)	0.66 (6)	≤ 0 (5)	0.02 (6)	0.34 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)
	Rand. CV	0.37 (5)	0.4 (5)	0.5 (5)	0.31 (5)	0.32 (5)	0.41 (6)	0.19 (5)	0.2 (6)	0.32 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
meanBase = 5	Uniform Demands	0.48 (2)	0.58 (2)	0.52 (2)	0.49 (2)	0.63 (2)	0.57 (2)	0.42 (2)	0.58 (2)	0.52 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)
	Const. CV	0.59 (2)	0.7 (2)	0.62 (2)	0.58 (2)	0.75 (2)	0.65 (2)	0.48 (2)	0.64 (2)	0.55 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.51 (2)	0.58 (2)	0.59 (2)	0.48 (2)	0.56 (2)	0.57 (2)	0.36 (2)	0.45 (2)	0.47 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.53 (1)	0.56 (1)	0.5 (1)	0.56 (1)	0.64 (1)	0.54 (1)	0.47 (1)	0.56 (1)	0.45 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.63 (1)	0.66 (1)	0.61 (1)	0.66 (1)	0.71 (1)	0.64 (1)	0.54 (1)	0.61 (1)	0.53 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.58 (1)	0.62 (1)	0.63 (1)	0.56 (1)	0.61 (1)	0.62 (1)	0.43 (1)	0.48 (1)	0.49 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 42: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.1 (5)	0.4 (6)	0.4 (6)	≤ 0 (5)	0.34 (5)	0.33 (6)	≤ 0 (6)	0.14 (6)	0.11 (6)	≤ 0 (5)	0.17 (6)	0.89 (6)	≤ 0 (5)	0.08 (6)	0.9 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)
	Const. CV	0.3 (5)	0.6 (5)	0.6 (6)	0.27 (5)	0.66 (5)	0.67 (6)	0.17 (5)	0.62 (5)	0.7 (6)	≤ 0 (5)	0.56 (6)	0.86 (6)	≤ 0 (6)	0.25 (6)	0.6 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)
	Rand. CV	0.39 (5)	0.44 (5)	0.52 (5)	0.42 (5)	0.49 (5)	0.59 (6)	0.47 (5)	0.55 (5)	0.67 (6)	0.43 (5)	0.44 (6)	0.74 (6)	0.31 (5)	0.26 (5)	0.43 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
meanBase = 5	Uniform Demands	0.22 (2)	0.46 (2)	0.41 (2)	0.06 (2)	0.35 (2)	0.32 (2)	≤ 0 (2)	0.08 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	0.23 (2)	≤ 0 (2)	≤ 0 (2)	0.16 (2)	≤ 0 (2)	≤ 0 (2)	0.03 (3)
	Const. CV	0.44 (2)	0.61 (2)	0.56 (2)	0.45 (2)	0.65 (2)	0.6 (2)	0.37 (2)	0.63 (2)	0.64 (2)	0.12 (2)	≤ 0 (2)	0.17 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.52 (2)	0.61 (2)	0.62 (2)	0.59 (2)	0.72 (2)	0.75 (2)	0.67 (2)	0.83 (2)	0.86 (2)	0.68 (2)	0.82 (2)	0.95 (2)	0.57 (2)	0.54 (2)	0.54 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.24 (1)	0.44 (1)	0.39 (1)	0.06 (1)	0.32 (1)	0.3 (1)	≤ 0 (1)	0.03 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.49 (1)	0.59 (1)	0.54 (1)	0.47 (1)	0.64 (1)	0.58 (1)	0.36 (1)	0.55 (1)	0.57 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.55 (1)	0.63 (1)	0.64 (1)	0.63 (1)	0.74 (1)	0.76 (1)	0.71 (1)	0.85 (1)	0.88 (1)	0.71 (1)	0.78 (1)	0.85 (1)	0.6 (1)	0.5 (1)	0.39 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 43: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.37 (6)	0.63 (6)	0.58 (6)	0.35 (6)	0.62 (6)	0.68 (7)	0.25 (6)	0.55 (7)	0.65 (7)	≤ 0 (6)	≤ 0 (6)	0.31 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)
	Const. CV	0.39 (6)	0.6 (7)	0.61 (6)	0.36 (6)	0.66 (6)	0.58 (7)	0.26 (6)	0.53 (7)	0.51 (7)	≤ 0 (6)	0.01 (6)	0.05 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)
	Rand. CV	0.22 (6)	0.08 (6)	0.11 (7)	0.17 (6)	0.02 (6)	0.03 (6)	0.07 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	0.54 (2)	0.62 (2)	0.58 (2)	0.55 (2)	0.67 (2)	0.64 (2)	0.47 (2)	0.63 (2)	0.59 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.55 (2)	0.66 (2)	0.6 (2)	0.56 (2)	0.7 (2)	0.63 (2)	0.46 (2)	0.65 (2)	0.55 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.25 (2)	0.19 (2)	0.13 (2)	0.19 (2)	0.13 (2)	0.05 (2)	0.07 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.58 (1)	0.6 (1)	0.56 (1)	0.62 (1)	0.65 (1)	0.58 (1)	0.52 (1)	0.57 (1)	0.53 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.6 (1)	0.66 (1)	0.6 (1)	0.62 (1)	0.68 (1)	0.63 (1)	0.52 (1)	0.6 (1)	0.55 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.29 (1)	0.18 (1)	0.14 (1)	0.25 (1)	0.11 (1)	0.05 (1)	0.11 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 44: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.28 (6)	0.55 (6)	0.52 (6)	0.26 (6)	0.54 (6)	0.61 (7)	0.17 (6)	0.5 (6)	0.6 (6)	≤ 0 (6)	0.17 (6)	0.58 (7)	≤ 0 (6)	0.09 (7)	0.53 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)
	Const. CV	0.45 (6)	0.67 (6)	0.62 (6)	0.57 (6)	0.8 (7)	0.82 (6)	0.71 (6)	1.03 (7)	1.09 (7)	0.57 (6)	1.19 (7)	1.82 (6)	0.36 (6)	0.7 (7)	1.1 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)
	Rand. CV	≤ 0 (6)	≤ 0 (6)	0.01 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	0.01 (6)	≤ 0 (6)	0.06 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	0.4 (2)	0.54 (2)	0.52 (2)	0.41 (2)	0.57 (2)	0.57 (2)	0.32 (2)	0.55 (2)	0.56 (2)	0.08 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.61 (2)	0.68 (2)	0.63 (2)	0.79 (2)	0.91 (2)	0.83 (2)	0.99 (2)	1.19 (2)	1.1 (2)	0.98 (2)	1.26 (2)	1.44 (3)	0.74 (2)	0.44 (2)	0.6 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	0.02 (2)	0.03 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.44 (1)	0.53 (1)	0.49 (1)	0.45 (1)	0.58 (1)	0.54 (1)	0.34 (1)	0.53 (1)	0.53 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.65 (1)	0.67 (1)	0.63 (1)	0.84 (1)	0.9 (1)	0.84 (1)	1.05 (1)	1.18 (1)	1.16 (1)	1.04 (1)	1.16 (1)	1.49 (2)	0.79 (1)	0.46 (1)	0.49 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0 (1)	0.01 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	0.09 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 45: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.45 (7)	0.64 (8)	0.65 (8)	0.41 (7)	0.73 (8)	0.68 (8)	0.31 (7)	0.64 (8)	0.67 (8)	≤ 0 (8)	≤ 0 (9)	0.17 (9)	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)
	Const. CV	0.11 (8)	0.22 (8)	0.25 (8)	0.05 (7)	0.23 (8)	0.2 (8)	≤ 0 (7)	0.06 (8)	0.13 (8)	≤ 0 (8)	≤ 0 (9)	0.15 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)
	Rand. CV	0.27 (7)	0.19 (8)	0.28 (8)	0.23 (7)	0.18 (7)	0.25 (8)	0.12 (7)	0.09 (7)	0.09 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.62 (3)	0.67 (3)	0.62 (3)	0.68 (3)	0.74 (3)	0.66 (3)	0.59 (3)	0.72 (3)	0.63 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	0.19 (3)	0.25 (3)	0.18 (3)	0.15 (3)	0.22 (3)	0.13 (3)	≤ 0 (3)	0.06 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	0.24 (3)	0.31 (3)	0.34 (3)	0.22 (3)	0.32 (3)	0.36 (3)	0.12 (3)	0.22 (3)	0.28 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.63 (2)	0.67 (2)	0.6 (2)	0.67 (2)	0.75 (2)	0.65 (2)	0.6 (2)	0.72 (2)	0.6 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.21 (2)	0.22 (2)	0.16 (2)	0.16 (2)	0.17 (2)	0.09 (2)	0.01 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.25 (2)	0.34 (2)	0.35 (2)	0.24 (2)	0.35 (2)	0.37 (2)	0.14 (2)	0.27 (2)	0.3 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 46: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.52 (7)	0.67 (8)	0.68 (8)	0.65 (7)	0.9 (8)	0.87 (8)	0.78 (7)	1.17 (8)	1.22 (8)	0.68 (8)	1.34 (8)	2.06 (8)	0.53 (7)	0.85 (8)	1.72 (9)	≤ 0 (8)	≤ 0 (8)	0.34 (8)
	Const. CV	≤ 0 (8)	0.19 (8)	0.25 (8)	≤ 0 (8)	0.12 (8)	0.14 (8)	≤ 0 (7)	0 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (9)	0.34 (8)	≤ 0 (8)	≤ 0 (9)	0.5 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)
	Rand. CV	0.28 (7)	0.26 (7)	0.3 (8)	0.26 (7)	0.31 (8)	0.33 (7)	0.3 (7)	0.37 (7)	0.47 (8)	0.22 (8)	0.11 (8)	0.22 (8)	0.11 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.68 (3)	0.71 (3)	0.65 (3)	0.87 (3)	0.97 (3)	0.9 (3)	1.1 (3)	1.29 (3)	1.25 (3)	1.04 (3)	1.45 (3)	1.83 (3)	0.87 (3)	0.98 (3)	1.32 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	0.02 (3)	0.2 (3)	0.16 (3)	≤ 0 (3)	0.11 (3)	0.07 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	0.31 (3)	0.35 (3)	0.35 (3)	0.35 (3)	0.52 (3)	0.51 (3)	0.34 (3)	0.58 (3)	0.65 (3)	0.3 (3)	0.37 (3)	0.41 (3)	0.12 (3)	0.06 (3)	0.03 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.7 (2)	0.71 (2)	0.64 (2)	0.93 (2)	0.99 (2)	0.89 (2)	1.15 (2)	1.34 (2)	1.26 (2)	1.2 (2)	1.54 (2)	1.65 (2)	1.05 (2)	0.83 (2)	0.9 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.05 (2)	0.17 (2)	0.14 (2)	≤ 0 (2)	0.05 (2)	0.03 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.04 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.34 (2)	0.37 (2)	0.37 (2)	0.38 (2)	0.51 (2)	0.54 (2)	0.38 (2)	0.62 (2)	0.68 (2)	0.34 (2)	0.39 (2)	0.37 (2)	0.15 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 47: Results for $L = 3$, $\ell = 2$, overPenalty = 0.5, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.22 (4)	0.46 (4)	0.46 (4)	0.2 (4)	0.46 (4)	0.54 (4)	0.15 (4)	0.42 (4)	0.47 (4)	≤ 0 (4)	0.09 (5)	0.8 (4)	≤ 0 (4)	0.99 (4)	2.05 (5)	2.18 (4)	5.27 (4)	7.65 (5)
	Const. CV	0.3 (4)	0.54 (4)	0.56 (4)	0.27 (4)	0.51 (4)	0.61 (4)	0.18 (4)	0.46 (4)	0.54 (4)	≤ 0 (4)	0.01 (5)	0.73 (4)	≤ 0 (4)	1.06 (4)	2.33 (4)	2.11 (4)	5.28 (4)	7.54 (5)
	Rand. CV	0.31 (3)	0.3 (4)	0.34 (4)	0.24 (4)	0.23 (4)	0.29 (4)	0.23 (3)	0.13 (4)	0.17 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	0.03 (4)	2.38 (4)	7.52 (4)	6.33 (4)	38.52 (4)
meanBase = 5	Uniform Demands	0.42 (1)	0.47 (1)	0.45 (1)	0.42 (1)	0.5 (1)	0.49 (1)	0.31 (1)	0.39 (1)	0.37 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	0.82 (2)	3.59 (2)	2.71 (1)	4.21 (2)	4.83 (2)
	Const. CV	0.52 (1)	0.58 (1)	0.57 (1)	0.51 (1)	0.6 (1)	0.54 (1)	0.37 (1)	0.47 (1)	0.41 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	0.68 (2)	3.42 (2)	2.57 (1)	3.96 (2)	4.64 (2)
	Rand. CV	0.43 (1)	0.5 (1)	0.52 (1)	0.4 (1)	0.47 (1)	0.49 (1)	0.26 (1)	0.33 (1)	0.34 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.16 (1)	19.52 (1)	24.89 (1)	24.77 (1)
meanBase = 10	Uniform Demands	0.46 (1)	0.49 (1)	0.44 (1)	0.47 (1)	0.52 (1)	0.46 (1)	0.34 (1)	0.39 (1)	0.33 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	1.02 (1)	4.18 (1)	3.45 (1)	5.22 (1)	5.44 (1)
	Const. CV	0.55 (1)	0.59 (1)	0.54 (1)	0.55 (1)	0.61 (1)	0.55 (1)	0.39 (1)	0.46 (1)	0.39 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.83 (1)	4.17 (1)	3.26 (1)	4.76 (1)	5.26 (1)
	Rand. CV	0.51 (1)	0.57 (1)	0.57 (1)	0.49 (1)	0.54 (1)	0.55 (1)	0.35 (1)	0.4 (1)	0.41 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.21 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)

Table 48: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.14 (3)	0.36 (4)	0.38 (4)	0.05 (3)	0.23 (4)	0.34 (4)	≤ 0 (3)	0.19 (4)	0.21 (4)	0.11 (3)	3 (4)	4.37 (4)	1.85 (4)	7.06 (4)	11.51 (4)	2.18 (4)	5.27 (4)	7.65 (5)
	Const. CV	0.24 (4)	0.43 (4)	0.53 (4)	0.22 (4)	0.5 (4)	0.59 (4)	0.2 (3)	0.48 (4)	0.61 (4)	0.36 (4)	3.65 (5)	4.98 (4)	4.48 (3)	7.07 (4)	11.76 (4)	2.11 (4)	5.28 (4)	7.54 (5)
	Rand. CV	0.33 (3)	0.34 (4)	0.37 (4)	0.37 (3)	0.39 (4)	0.43 (4)	0.38 (3)	0.43 (4)	0.49 (4)	0.35 (4)	0.74 (4)	1.56 (4)	0.6 (3)	2.45 (4)	5.3 (4)	7.52 (4)	6.33 (4)	38.52 (4)
meanBase = 5	Uniform Demands	0.18 (1)	0.35 (1)	0.35 (1)	0.03 (1)	0.23 (1)	0.26 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (1)	≤ 0 (1)	1.83 (2)	5.85 (2)	2.23 (1)	24.11 (2)	25.23 (2)	2.71 (1)	4.21 (2)	4.83 (2)
	Const. CV	0.39 (1)	0.51 (1)	0.5 (1)	0.39 (1)	0.54 (1)	0.56 (1)	0.3 (1)	0.49 (1)	0.49 (1)	0.42 (1)	4 (2)	8.19 (2)	4.74 (1)	25.61 (2)	23.3 (2)	2.57 (1)	3.96 (2)	4.64 (2)
	Rand. CV	0.44 (1)	0.53 (1)	0.54 (1)	0.51 (1)	0.61 (1)	0.63 (1)	0.57 (1)	0.7 (1)	0.72 (1)	0.54 (1)	0.81 (1)	1.46 (1)	0.55 (1)	3.76 (1)	6.73 (1)	19.52 (1)	24.89 (1)	24.77 (1)
meanBase = 10	Uniform Demands	0.19 (1)	0.36 (1)	0.34 (1)	≤ 0 (1)	0.22 (1)	0.22 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	2.08 (1)	6.28 (1)	2.24 (1)	27.21 (1)	28.06 (1)	3.45 (1)	5.22 (1)	5.44 (1)
	Const. CV	0.42 (1)	0.51 (1)	0.48 (1)	0.39 (1)	0.54 (1)	0.5 (1)	0.28 (1)	0.43 (1)	0.42 (1)	0.05 (1)	5.07 (1)	9.52 (1)	6.6 (1)	29.03 (1)	26.79 (1)	3.26 (1)	4.76 (1)	5.26 (1)
	Rand. CV	0.49 (1)	0.56 (1)	0.58 (1)	0.56 (1)	0.67 (1)	0.68 (1)	0.63 (1)	0.76 (1)	0.79 (1)	0.63 (1)	0.67 (1)	0.68 (1)	0.51 (1)	0.38 (1)	8.39 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)

Table 49: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.38 (4)	0.46 (5)	0.64 (5)	0.28 (5)	0.53 (4)	0.61 (5)	0.21 (5)	0.35 (5)	0.56 (5)	≤ 0 (5)	0.1 (5)	0.68 (5)	≤ 0 (5)	0.83 (5)	1.91 (5)	58.47 (5)	46.17 (5)	42.83 (5)
	Const. CV	0.38 (4)	0.55 (5)	0.69 (5)	0.27 (5)	0.56 (4)	0.66 (5)	0.18 (5)	0.43 (5)	0.57 (5)	≤ 0 (5)	0.03 (4)	0.57 (5)	≤ 0 (5)	0.68 (5)	1.8 (5)	58.62 (5)	47.17 (5)	43.91 (5)
	Rand. CV	0.18 (5)	0.08 (4)	0.09 (5)	0.12 (5)	≤ 0 (5)	≤ 0 (5)	0.13 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	≤ 0 (5)	1.29 (5)	71.17 (4)	57.76 (5)	48.52 (5)
meanBase = 5	Uniform Demands	0.47 (1)	0.53 (1)	0.5 (1)	0.47 (1)	0.56 (1)	0.52 (1)	0.35 (1)	0.45 (1)	0.4 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	2.46 (2)	52.67 (1)	8.4 (2)	7.49 (2)
	Const. CV	0.48 (1)	0.55 (1)	0.54 (1)	0.46 (1)	0.55 (1)	0.54 (1)	0.34 (1)	0.43 (1)	0.42 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	2.37 (2)	53.51 (1)	9.25 (2)	8.03 (2)
	Rand. CV	0.19 (1)	0.09 (1)	0.07 (1)	0.13 (1)	0.04 (1)	≤ 0 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	28.03 (2)	54.34 (1)	15.43 (2)
meanBase = 10	Uniform Demands	0.51 (1)	0.56 (1)	0.52 (1)	0.53 (1)	0.58 (1)	0.53 (1)	0.42 (1)	0.48 (1)	0.44 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	2.75 (1)	59.99 (1)	25.7 (1)	14.17 (1)
	Const. CV	0.53 (1)	0.61 (1)	0.58 (1)	0.52 (1)	0.62 (1)	0.58 (1)	0.41 (1)	0.53 (1)	0.48 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	2.57 (1)	60.76 (1)	27.19 (1)	15.28 (1)
	Rand. CV	0.22 (1)	0.12 (1)	0.06 (1)	0.16 (1)	0.03 (1)	≤ 0 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	72.84 (1)	61.53 (1)	54.39 (1)

Table 50: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.29 (4)	0.39 (5)	0.58 (5)	0.28 (4)	0.45 (4)	0.53 (5)	0.2 (4)	0.42 (5)	0.53 (5)	≤ 0 (5)	0.42 (4)	1.52 (5)	≤ 0 (4)	0.74 (5)	2.07 (5)	58.47 (5)	46.17 (5)	42.83 (5)
	Const. CV	0.4 (4)	0.56 (5)	0.7 (5)	0.49 (4)	0.73 (5)	0.76 (5)	0.56 (5)	0.92 (5)	1.09 (5)	0.6 (4)	3.17 (5)	4.88 (5)	0.4 (5)	3.71 (5)	5.25 (5)	58.62 (5)	47.17 (5)	43.91 (5)
	Rand. CV	0.03 (4)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	0.04 (4)	≤ 0 (5)	≤ 0 (5)	0.16 (4)	0.23 (5)	0.92 (5)	0.34 (5)	3.07 (4)	5.97 (4)	71.17 (4)	57.76 (5)	48.52 (5)
meanBase = 5	Uniform Demands	0.34 (1)	0.45 (1)	0.43 (1)	0.34 (1)	0.47 (1)	0.44 (1)	0.27 (1)	0.41 (1)	0.4 (1)	0.03 (1)	≤ 0 (2)	0.63 (2)	≤ 0 (1)	≤ 0 (2)	0.88 (2)	52.67 (1)	8.4 (2)	7.49 (2)
	Const. CV	0.53 (1)	0.58 (1)	0.57 (1)	0.68 (1)	0.75 (1)	0.75 (1)	0.83 (1)	0.98 (1)	1.02 (2)	0.8 (1)	2.8 (2)	5.83 (2)	0.58 (1)	3.75 (2)	9.27 (2)	53.51 (1)	9.25 (2)	8.03 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (2)	0.2 (1)	0.61 (1)	0.49 (2)	3.37 (1)	6.61 (1)	28.03 (2)	54.34 (1)	15.43 (2)
meanBase = 10	Uniform Demands	0.38 (1)	0.47 (1)	0.45 (1)	0.37 (1)	0.51 (1)	0.48 (1)	0.31 (1)	0.49 (1)	0.44 (1)	0.04 (1)	≤ 0 (1)	0.55 (1)	≤ 0 (1)	≤ 0 (1)	0.77 (1)	59.99 (1)	25.7 (1)	14.17 (1)
	Const. CV	0.58 (1)	0.64 (1)	0.61 (1)	0.74 (1)	0.84 (1)	0.79 (1)	0.93 (1)	1.13 (1)	1.08 (1)	0.94 (1)	3.2 (1)	6.19 (1)	0.72 (1)	4.47 (1)	10.25 (1)	60.76 (1)	27.19 (1)	15.28 (1)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.05 (1)	0.25 (1)	0.73 (1)	0.63 (1)	4.19 (1)	8.76 (1)	72.84 (1)	61.53 (1)	54.39 (1)

Table 51: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.34 (5)	0.64 (5)	0.59 (6)	0.31 (5)	0.6 (6)	0.58 (6)	0.2 (5)	0.49 (6)	0.54 (6)	≤ 0 (5)	0.13 (6)	0.22 (6)	≤ 0 (5)	0.64 (6)	1.68 (6)	77.08 (5)	61.71 (6)	52.43 (6)
	Const. CV	0.03 (5)	0.21 (6)	0.25 (6)	≤ 0 (5)	0.2 (6)	0.12 (6)	≤ 0 (5)	0.09 (6)	0.03 (6)	≤ 0 (5)	0.1 (6)	0.25 (6)	≤ 0 (5)	1.06 (6)	1.57 (6)	73.9 (5)	58.1 (6)	56.34 (6)
	Rand. CV	0.16 (5)	0.11 (5)	0.18 (5)	0.12 (5)	0.06 (5)	0.16 (6)	0.03 (5)	≤ 0 (5)	0.05 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	76.17 (5)	74.14 (5)	67.76 (6)
meanBase = 5	Uniform Demands	0.52 (2)	0.61 (2)	0.55 (2)	0.53 (2)	0.65 (2)	0.62 (2)	0.43 (2)	0.6 (2)	0.53 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	0 (2)	2.16 (3)	69.67 (2)	54.13 (2)	24.99 (3)
	Const. CV	0.14 (2)	0.19 (2)	0.16 (2)	0.09 (2)	0.14 (2)	0.06 (2)	≤ 0 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	0.15 (2)	2.32 (3)	66.18 (2)	50.58 (2)	24.54 (3)
	Rand. CV	0.19 (2)	0.27 (2)	0.32 (2)	0.16 (2)	0.27 (2)	0.33 (2)	0.03 (2)	0.17 (2)	0.26 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	71.97 (2)	66.83 (2)	63.28 (2)
meanBase = 10	Uniform Demands	0.56 (1)	0.61 (1)	0.54 (1)	0.59 (1)	0.66 (1)	0.56 (1)	0.49 (1)	0.58 (1)	0.46 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	1.62 (2)	54.53 (1)	16.13 (2)	12.64 (2)
	Const. CV	0.17 (1)	0.21 (1)	0.11 (1)	0.12 (1)	0.14 (1)	0.02 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	1.19 (2)	50.87 (1)	15.35 (2)	12.13 (2)
	Rand. CV	0.22 (1)	0.28 (1)	0.3 (1)	0.19 (1)	0.27 (1)	0.3 (1)	0.08 (1)	0.16 (1)	0.18 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	61.88 (1)	52.29 (1)	45.96 (1)

Table 52: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.41 (6)	0.66 (5)	0.62 (6)	0.52 (6)	0.77 (6)	0.75 (6)	0.64 (5)	0.99 (6)	1.1 (6)	0.67 (6)	2.73 (6)	4.19 (6)	2.43 (5)	7.51 (6)	9.66 (6)	77.08 (5)	61.71 (6)	52.43 (6)
	Const. CV	≤ 0 (5)	0.18 (6)	0.24 (6)	≤ 0 (5)	0.13 (6)	0.07 (6)	≤ 0 (6)	0.07 (6)	0.14 (6)	0.07 (5)	1.35 (6)	2.1 (6)	0.59 (6)	4.17 (6)	5.53 (6)	73.9 (5)	58.1 (6)	56.34 (6)
	Rand. CV	0.19 (5)	0.14 (6)	0.2 (6)	0.23 (5)	0.19 (6)	0.27 (5)	0.19 (5)	0.19 (6)	0.35 (5)	0.14 (5)	0.53 (5)	0.73 (6)	0.63 (5)	1.62 (6)	3.27 (5)	76.17 (5)	74.14 (5)	67.76 (6)
meanBase = 5	Uniform Demands	0.6 (2)	0.66 (2)	0.61 (2)	0.78 (2)	0.88 (2)	0.82 (2)	0.98 (2)	1.18 (2)	1.17 (2)	1.1 (2)	2.91 (2)	5.07 (2)	3.17 (2)	15.35 (2)	19.4 (2)	69.67 (2)	54.13 (2)	24.99 (3)
	Const. CV	0 (2)	0.15 (2)	0.12 (2)	≤ 0 (2)	0.04 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.1 (2)	0.78 (2)	1.43 (2)	1.19 (2)	7.19 (2)	10.4 (3)	66.18 (2)	50.58 (2)	24.54 (3)
	Rand. CV	0.26 (2)	0.31 (2)	0.33 (2)	0.3 (2)	0.46 (2)	0.47 (2)	0.29 (2)	0.46 (2)	0.58 (2)	0.25 (2)	0.66 (2)	1.17 (2)	0.61 (2)	2.94 (2)	5.21 (2)	71.97 (2)	66.83 (2)	63.28 (2)
meanBase = 10	Uniform Demands	0.63 (1)	0.63 (1)	0.57 (1)	0.83 (1)	0.87 (1)	0.82 (1)	1.03 (1)	1.19 (1)	1.16 (1)	1.16 (1)	2.65 (1)	4.29 (2)	2.91 (1)	18.13 (1)	25.22 (2)	54.53 (1)	16.13 (2)	12.64 (2)
	Const. CV	0.03 (1)	0.17 (1)	0.09 (1)	≤ 0 (1)	0.05 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.41 (1)	1.13 (2)	1.13 (1)	6.69 (1)	13.25 (2)	50.87 (1)	15.35 (2)	12.13 (2)
	Rand. CV	0.29 (1)	0.31 (1)	0.31 (1)	0.33 (1)	0.43 (1)	0.44 (1)	0.33 (1)	0.49 (1)	0.56 (1)	0.29 (1)	0.59 (1)	0.99 (1)	0.6 (1)	3.63 (1)	6.08 (1)	61.88 (1)	52.29 (1)	45.96 (1)

Table 53: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

B Results for Numerical Study II

This appendix contains all 48 tables of results for our second numerical study, *including* the four tables provided in the appendix to the paper.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.44 (8)	0.62 (8)	0.58 (8)	0.39 (7)	0.65 (8)	0.67 (8)	0.31 (7)	0.63 (8)	0.68 (8)	≤ 0 (8)	≤ 0 (8)	0.3 (9)	≤ 0 (8)	≤ 0 (8)	0.09 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (8)
	Const. CV	0.48 (7)	0.72 (8)	0.68 (8)	0.46 (7)	0.74 (8)	0.75 (8)	0.33 (7)	0.72 (8)	0.71 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
	Rand. CV	0.43 (7)	0.53 (8)	0.56 (7)	0.43 (7)	0.42 (7)	0.64 (8)	0.31 (7)	0.3 (7)	0.49 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.6 (3)	0.63 (3)	0.57 (3)	0.64 (3)	0.71 (3)	0.64 (3)	0.58 (3)	0.69 (3)	0.61 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.69 (3)	0.73 (3)	0.69 (3)	0.74 (3)	0.79 (3)	0.73 (3)	0.67 (3)	0.72 (3)	0.68 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.59 (3)	0.66 (3)	0.69 (3)	0.58 (3)	0.67 (3)	0.67 (3)	0.48 (3)	0.56 (3)	0.6 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.6 (2)	0.63 (2)	0.56 (2)	0.66 (2)	0.71 (2)	0.62 (2)	0.6 (2)	0.69 (2)	0.59 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.7 (2)	0.72 (2)	0.69 (2)	0.75 (2)	0.79 (2)	0.75 (2)	0.68 (2)	0.73 (2)	0.68 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.64 (2)	0.68 (2)	0.69 (2)	0.65 (2)	0.67 (2)	0.69 (2)	0.54 (2)	0.58 (2)	0.59 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 54: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.18 (7)	0.49 (8)	0.48 (8)	0.05 (8)	0.42 (8)	0.44 (8)	≤ 0 (7)	0.21 (8)	0.19 (8)	≤ 0 (8)	0.04 (8)	0.33 (8)	≤ 0 (7)	≤ 0 (8)	0.43 (9)	≤ 0 (8)	≤ 0 (8)	0.09 (8)
	Const. CV	0.38 (7)	0.66 (8)	0.63 (8)	0.37 (7)	0.69 (8)	0.71 (8)	0.32 (8)	0.64 (8)	0.72 (8)	0.06 (7)	0.4 (9)	0.28 (8)	≤ 0 (8)	0.02 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
	Rand. CV	0.47 (8)	0.5 (7)	0.58 (7)	0.51 (7)	0.58 (7)	0.76 (8)	0.6 (8)	0.71 (8)	0.87 (8)	0.58 (8)	0.64 (8)	0.95 (8)	0.45 (7)	0.34 (8)	0.52 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.3 (3)	0.47 (3)	0.48 (3)	0.13 (3)	0.38 (3)	0.38 (3)	0.03 (3)	0.12 (3)	0.12 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.56 (3)	0.64 (3)	0.61 (3)	0.58 (3)	0.68 (3)	0.66 (3)	0.43 (3)	0.66 (3)	0.71 (3)	0.17 (3)	0.01 (3)	0.1 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.56 (3)	0.67 (3)	0.68 (3)	0.65 (3)	0.78 (3)	0.81 (3)	0.74 (3)	0.91 (3)	0.95 (3)	0.75 (3)	0.94 (3)	1.03 (3)	0.64 (3)	0.63 (3)	0.5 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.28 (2)	0.49 (2)	0.45 (2)	0.09 (2)	0.39 (2)	0.36 (2)	≤ 0 (2)	0.09 (2)	0.07 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.55 (2)	0.63 (2)	0.62 (2)	0.58 (2)	0.68 (2)	0.67 (2)	0.46 (2)	0.66 (2)	0.67 (2)	0.16 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.6 (2)	0.69 (2)	0.69 (2)	0.7 (2)	0.81 (2)	0.83 (2)	0.8 (2)	0.95 (2)	0.96 (2)	0.85 (2)	0.99 (2)	1.1 (2)	0.74 (2)	0.67 (2)	0.53 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 55: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (8)	≤ 0 (8)	0.09 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (8)
	Const. CV	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
	Rand. CV	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 56: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, uniform costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (8)	≤ 0 (9)	0.1 (8)	≤ 0 (8)	≤ 0 (8)	0.35 (9)	≤ 0 (8)	≤ 0 (8)	0.09 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (8)
	Const. CV	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
	Rand. CV	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 57: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, uniform costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.48 (8)	0.63 (9)	0.65 (9)	0.45 (8)	0.7 (8)	0.68 (9)	0.33 (8)	0.61 (9)	0.64 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)
	Const. CV	0.48 (8)	0.63 (9)	0.58 (9)	0.45 (8)	0.69 (8)	0.63 (9)	0.33 (8)	0.6 (9)	0.7 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)
	Rand. CV	0.28 (8)	0.15 (8)	0.18 (8)	0.23 (8)	0.07 (8)	0.12 (8)	0.12 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)
meanBase = 5	Uniform Demands	0.64 (3)	0.66 (3)	0.6 (3)	0.68 (3)	0.75 (3)	0.66 (3)	0.62 (3)	0.7 (3)	0.62 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	0.6 (3)	0.65 (3)	0.62 (3)	0.61 (3)	0.75 (3)	0.65 (3)	0.57 (3)	0.72 (3)	0.59 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	0.3 (3)	0.22 (3)	0.17 (3)	0.26 (3)	0.16 (3)	0.11 (3)	0.15 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.63 (2)	0.67 (2)	0.61 (2)	0.7 (2)	0.75 (2)	0.67 (2)	0.63 (2)	0.73 (2)	0.64 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.62 (2)	0.68 (2)	0.62 (2)	0.7 (2)	0.72 (2)	0.66 (2)	0.58 (2)	0.68 (2)	0.59 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.32 (2)	0.21 (2)	0.16 (2)	0.27 (2)	0.15 (2)	0.09 (2)	0.15 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 58: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.37 (8)	0.55 (9)	0.58 (9)	0.38 (8)	0.63 (9)	0.62 (9)	0.27 (8)	0.61 (9)	0.64 (9)	0.01 (8)	0.15 (9)	0.31 (9)	≤ 0 (9)	0.03 (10)	0.11 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)
	Const. CV	0.52 (8)	0.7 (9)	0.7 (9)	0.67 (8)	0.87 (9)	0.81 (9)	0.82 (8)	1.14 (9)	1.25 (9)	0.74 (8)	1.31 (9)	1.28 (9)	0.52 (8)	0.75 (9)	0.7 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)
	Rand. CV	≤ 0 (8)	≤ 0 (8)	0.06 (8)	0.03 (8)	≤ 0 (8)	≤ 0 (8)	0.02 (8)	≤ 0 (8)	≤ 0 (8)	0.07 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)
meanBase = 5	Uniform Demands	0.49 (3)	0.57 (3)	0.55 (3)	0.51 (3)	0.65 (3)	0.58 (3)	0.41 (3)	0.65 (3)	0.61 (3)	0.14 (3)	0.11 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	0.67 (3)	0.72 (3)	0.65 (3)	0.85 (3)	0.96 (3)	0.88 (3)	1.07 (3)	1.29 (3)	1.22 (3)	1.07 (3)	1.39 (3)	1.45 (4)	0.82 (3)	0.73 (3)	0.51 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	≤ 0 (3)	0.02 (3)	0.04 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	0.04 (3)	≤ 0 (3)	≤ 0 (3)	0.09 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.5 (2)	0.58 (2)	0.54 (2)	0.49 (2)	0.66 (2)	0.6 (2)	0.38 (2)	0.62 (2)	0.59 (2)	0.1 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.7 (2)	0.7 (2)	0.65 (2)	0.9 (2)	0.96 (2)	0.88 (2)	1.15 (2)	1.3 (2)	1.25 (2)	1.24 (2)	1.43 (2)	1.41 (2)	1 (2)	0.67 (2)	0.46 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	0.03 (2)	0.03 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	0.12 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 59: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)
	Const. CV	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)
	Rand. CV	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)
meanBase = 5	Uniform Demands	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 60: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, proportional costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)	≤ 0 (8)	≤ 0 (9)	0.07 (9)
	Const. CV	≤ 0 (8)	0.06 (9)	0.98 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)	≤ 0 (8)	≤ 0 (9)	≤ 0 (9)
	Rand. CV	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)	≤ 0 (8)	≤ 0 (8)	≤ 0 (9)
meanBase = 5	Uniform Demands	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	≤ 0 (3)	≤ 0 (3)	0.39 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (2)	≤ 0 (2)	0.25 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 61: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, proportional costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		`chiBase` = 0			`chiBase` = 1			`chiBase` = 2			`chiBase` = 5			`chiBase` = 7			`chiBase` = 999		
	`CVBase` →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
`meanBase` = 1	Uniform Demands	0.5 (11)	0.65 (12)	0.64 (12)	0.49 (11)	0.74 (11)	0.68 (12)	0.4 (11)	0.67 (11)	0.67 (12)	≤ 0 (11)	≤ 0 (11)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)
	Const. CV	0.15 (11)	0.27 (11)	0.2 (12)	0.09 (11)	0.26 (12)	0.19 (12)	≤ 0 (11)	0.13 (11)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)
	Rand. CV	0.2 (11)	0.24 (11)	0.31 (11)	0.24 (11)	0.22 (11)	0.31 (11)	0.13 (11)	0.11 (11)	0.24 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)
`meanBase` = 5	Uniform Demands	0.63 (4)	0.68 (4)	0.63 (4)	0.68 (4)	0.76 (4)	0.71 (4)	0.63 (4)	0.74 (4)	0.68 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)
	Const. CV	0.24 (4)	0.21 (4)	0.17 (4)	0.2 (4)	0.16 (4)	0.11 (4)	0.06 (4)	0.02 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)
	Rand. CV	0.29 (4)	0.33 (4)	0.36 (4)	0.28 (4)	0.35 (4)	0.39 (4)	0.19 (4)	0.25 (4)	0.32 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)
`meanBase` = 10	Uniform Demands	0.68 (3)	0.68 (3)	0.66 (3)	0.76 (3)	0.77 (3)	0.74 (3)	0.74 (3)	0.73 (3)	0.74 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.23 (3)	0.26 (3)	0.2 (3)	0.19 (3)	0.23 (3)	0.16 (3)	0.05 (3)	0.1 (3)	0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.3 (3)	0.35 (3)	0.41 (3)	0.29 (3)	0.39 (3)	0.44 (3)	0.19 (3)	0.28 (3)	0.35 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)

Table 62: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.57 (11)	0.68 (12)	0.68 (12)	0.74 (11)	0.94 (11)	0.89 (12)	0.92 (11)	1.25 (11)	1.25 (12)	0.83 (11)	1.3 (11)	1.59 (12)	0.64 (11)	0.82 (11)	0.89 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)
	Const. CV	0 (11)	0.22 (11)	0.18 (12)	≤ 0 (11)	0.15 (11)	0.12 (12)	≤ 0 (11)	0 (11)	≤ 0 (12)	≤ 0 (11)	0.06 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)
	Rand. CV	0.3 (11)	0.27 (11)	0.32 (11)	0.29 (11)	0.37 (11)	0.45 (11)	0.31 (11)	0.4 (11)	0.52 (11)	0.22 (11)	0.24 (11)	0.23 (11)	0.08 (11)	≤ 0 (11)	≤ 0 (12)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)
meanBase = 5	Uniform Demands	0.71 (4)	0.72 (4)	0.67 (4)	0.94 (4)	1 (4)	0.95 (4)	1.17 (4)	1.35 (4)	1.33 (4)	1.2 (4)	1.52 (4)	1.42 (5)	1.04 (4)	0.97 (4)	0.91 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)
	Const. CV	0.05 (4)	0.17 (4)	0.15 (4)	≤ 0 (4)	0.05 (4)	0.03 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	0.02 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)
	Rand. CV	0.36 (4)	0.37 (4)	0.38 (4)	0.39 (4)	0.52 (4)	0.55 (4)	0.41 (4)	0.61 (4)	0.73 (4)	0.37 (4)	0.4 (4)	0.46 (4)	0.17 (4)	0.04 (4)	0.03 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)
meanBase = 10	Uniform Demands	0.74 (3)	0.72 (3)	0.7 (3)	0.97 (3)	1 (3)	0.97 (3)	1.23 (3)	1.41 (3)	1.37 (3)	1.26 (3)	1.57 (3)	1.61 (3)	1.19 (3)	1 (3)	1.03 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.05 (3)	0.21 (3)	0.18 (3)	≤ 0 (3)	0.1 (3)	0.08 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	0.03 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	0.37 (3)	0.39 (3)	0.42 (3)	0.42 (3)	0.55 (3)	0.56 (3)	0.44 (3)	0.65 (3)	0.73 (3)	0.4 (3)	0.5 (3)	0.4 (3)	0.18 (3)	0.09 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)

Table 63: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)
	Const. CV	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)
	Rand. CV	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)
meanBase = 5	Uniform Demands	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)
	Const. CV	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)
	Rand. CV	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)
meanBase = 10	Uniform Demands	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)

Table 64: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, random costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (11)	0.07 (12)	0.7 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)
	Const. CV	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)	≤ 0 (11)	≤ 0 (12)	≤ 0 (12)
	Rand. CV	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)	≤ 0 (11)
meanBase = 5	Uniform Demands	≤ 0 (4)	≤ 0 (4)	0.75 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)
	Const. CV	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)
	Rand. CV	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)
meanBase = 10	Uniform Demands	≤ 0 (3)	≤ 0 (3)	0.78 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Const. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
	Rand. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)

Table 65: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.25`, random costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.3 (5)	0.52 (6)	0.49 (6)	0.29 (5)	0.62 (5)	0.56 (6)	0.19 (5)	0.49 (6)	0.53 (6)	≤ 0 (5)	0.2 (6)	0.32 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)
	Const. CV	0.39 (5)	0.6 (6)	0.65 (6)	0.36 (5)	0.72 (5)	0.71 (6)	0.23 (5)	0.53 (6)	0.66 (6)	≤ 0 (5)	0.02 (6)	0.34 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)
	Rand. CV	0.37 (5)	0.4 (5)	0.5 (5)	0.31 (5)	0.32 (5)	0.41 (6)	0.19 (5)	0.2 (6)	0.32 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
meanBase = 5	Uniform Demands	0.48 (2)	0.58 (2)	0.52 (2)	0.49 (2)	0.63 (2)	0.57 (2)	0.42 (2)	0.58 (2)	0.52 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)
	Const. CV	0.59 (2)	0.7 (2)	0.62 (2)	0.58 (2)	0.75 (2)	0.65 (2)	0.48 (2)	0.64 (2)	0.55 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.51 (2)	0.58 (2)	0.59 (2)	0.48 (2)	0.56 (2)	0.57 (2)	0.36 (2)	0.45 (2)	0.47 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.53 (1)	0.56 (1)	0.5 (1)	0.56 (1)	0.64 (1)	0.54 (1)	0.47 (1)	0.56 (1)	0.45 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.63 (1)	0.66 (1)	0.61 (1)	0.66 (1)	0.71 (1)	0.64 (1)	0.54 (1)	0.61 (1)	0.53 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.58 (1)	0.62 (1)	0.63 (1)	0.56 (1)	0.61 (1)	0.62 (1)	0.43 (1)	0.48 (1)	0.49 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 66: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.1 (5)	0.4 (6)	0.4 (6)	≤ 0 (5)	0.34 (5)	0.33 (6)	≤ 0 (6)	0.14 (6)	0.11 (6)	≤ 0 (5)	0.17 (6)	0.89 (6)	≤ 0 (5)	0.08 (6)	0.9 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)
	Const. CV	0.3 (5)	0.6 (5)	0.6 (6)	0.27 (5)	0.66 (5)	0.67 (6)	0.17 (5)	0.62 (5)	0.7 (6)	≤ 0 (5)	0.56 (6)	0.86 (6)	≤ 0 (6)	0.25 (6)	0.6 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)
	Rand. CV	0.39 (5)	0.44 (5)	0.52 (5)	0.42 (5)	0.49 (5)	0.59 (6)	0.47 (5)	0.55 (5)	0.67 (6)	0.43 (5)	0.44 (6)	0.74 (6)	0.31 (5)	0.26 (5)	0.43 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
meanBase = 5	Uniform Demands	0.22 (2)	0.46 (2)	0.41 (2)	0.06 (2)	0.35 (2)	0.32 (2)	≤ 0 (2)	0.08 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	0.23 (2)	≤ 0 (2)	≤ 0 (2)	0.16 (2)	≤ 0 (2)	≤ 0 (2)	0.03 (3)
	Const. CV	0.44 (2)	0.61 (2)	0.56 (2)	0.45 (2)	0.65 (2)	0.6 (2)	0.37 (2)	0.63 (2)	0.64 (2)	0.12 (2)	≤ 0 (2)	0.17 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.52 (2)	0.61 (2)	0.62 (2)	0.59 (2)	0.72 (2)	0.75 (2)	0.67 (2)	0.83 (2)	0.86 (2)	0.68 (2)	0.82 (2)	0.95 (2)	0.57 (2)	0.54 (2)	0.54 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.24 (1)	0.44 (1)	0.39 (1)	0.06 (1)	0.32 (1)	0.3 (1)	≤ 0 (1)	0.03 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	0.49 (1)	0.59 (1)	0.54 (1)	0.47 (1)	0.64 (1)	0.58 (1)	0.36 (1)	0.55 (1)	0.57 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	0.55 (1)	0.63 (1)	0.64 (1)	0.63 (1)	0.74 (1)	0.76 (1)	0.71 (1)	0.85 (1)	0.88 (1)	0.71 (1)	0.78 (1)	0.85 (1)	0.6 (1)	0.5 (1)	0.39 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 67: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)
	Const. CV	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)
	Rand. CV	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
meanBase = 5	Uniform Demands	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)
	Const. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 68: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, uniform costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (5)	0.22 (6)	0.94 (6)	≤ 0 (5)	0.06 (6)	0.64 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)	≤ 0 (5)	0.04 (6)	0.77 (6)
	Const. CV	≤ 0 (5)	0.07 (6)	0.57 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)	≤ 0 (5)	0.15 (6)	0.77 (6)
	Rand. CV	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)
meanBase = 5	Uniform Demands	≤ 0 (2)	≤ 0 (2)	0.15 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)	≤ 0 (2)	≤ 0 (2)	0.03 (3)
	Const. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Const. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 69: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, uniform costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.37 (6)	0.63 (6)	0.58 (6)	0.35 (6)	0.62 (6)	0.68 (7)	0.25 (6)	0.55 (7)	0.65 (7)	≤ 0 (6)	≤ 0 (6)	0.31 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)
	Const. CV	0.39 (6)	0.6 (7)	0.61 (6)	0.36 (6)	0.66 (6)	0.58 (7)	0.26 (6)	0.53 (7)	0.51 (7)	≤ 0 (6)	0.01 (6)	0.05 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)
	Rand. CV	0.22 (6)	0.08 (6)	0.11 (7)	0.17 (6)	0.02 (6)	0.03 (6)	0.07 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	0.54 (2)	0.62 (2)	0.58 (2)	0.55 (2)	0.67 (2)	0.64 (2)	0.47 (2)	0.63 (2)	0.59 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.55 (2)	0.66 (2)	0.6 (2)	0.56 (2)	0.7 (2)	0.63 (2)	0.46 (2)	0.65 (2)	0.55 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	0.25 (2)	0.19 (2)	0.13 (2)	0.19 (2)	0.13 (2)	0.05 (2)	0.07 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.58 (1)	0.6 (1)	0.56 (1)	0.62 (1)	0.65 (1)	0.58 (1)	0.52 (1)	0.57 (1)	0.53 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.6 (1)	0.66 (1)	0.6 (1)	0.62 (1)	0.68 (1)	0.63 (1)	0.52 (1)	0.6 (1)	0.55 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.29 (1)	0.18 (1)	0.14 (1)	0.25 (1)	0.11 (1)	0.05 (1)	0.11 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 70: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.28 (6)	0.55 (6)	0.52 (6)	0.26 (6)	0.54 (6)	0.61 (7)	0.17 (6)	0.5 (6)	0.6 (6)	≤ 0 (6)	0.17 (6)	0.58 (7)	≤ 0 (6)	0.09 (7)	0.53 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)
	Const. CV	0.45 (6)	0.67 (6)	0.62 (6)	0.57 (6)	0.8 (7)	0.82 (6)	0.71 (6)	1.03 (7)	1.09 (7)	0.57 (6)	1.19 (7)	1.82 (6)	0.36 (6)	0.7 (7)	1.1 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)
	Rand. CV	≤ 0 (6)	≤ 0 (6)	0.01 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	0.01 (6)	≤ 0 (6)	0.06 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	0.4 (2)	0.54 (2)	0.52 (2)	0.41 (2)	0.57 (2)	0.57 (2)	0.32 (2)	0.55 (2)	0.56 (2)	0.08 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)
	Const. CV	0.61 (2)	0.68 (2)	0.63 (2)	0.79 (2)	0.91 (2)	0.83 (2)	0.99 (2)	1.19 (2)	1.1 (2)	0.98 (2)	1.26 (2)	1.44 (3)	0.74 (2)	0.44 (2)	0.6 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	0.02 (2)	0.03 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.06 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.44 (1)	0.53 (1)	0.49 (1)	0.45 (1)	0.58 (1)	0.54 (1)	0.34 (1)	0.53 (1)	0.53 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.65 (1)	0.67 (1)	0.63 (1)	0.84 (1)	0.9 (1)	0.84 (1)	1.05 (1)	1.18 (1)	1.16 (1)	1.04 (1)	1.16 (1)	1.49 (2)	0.79 (1)	0.46 (1)	0.49 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0 (1)	0.01 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	0.09 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 71: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)
	Const. CV	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)
	Rand. CV	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)
	Const. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 72: Results for $L = 3$, $\ell = 2$, overPenalty = 0.5, proportional costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (6)	≤ 0 (6)	0.52 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)	≤ 0 (6)	0.04 (6)	0.32 (7)
	Const. CV	≤ 0 (6)	0.27 (6)	1.01 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)	≤ 0 (6)	≤ 0 (7)	0.31 (7)
	Rand. CV	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)
	Const. CV	≤ 0 (2)	≤ 0 (2)	0.35 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (1)	≤ 0 (2)	0.25 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 73: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, proportional costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.45 (7)	0.64 (8)	0.65 (8)	0.41 (7)	0.73 (8)	0.68 (8)	0.31 (7)	0.64 (8)	0.67 (8)	≤ 0 (8)	≤ 0 (9)	0.17 (9)	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)
	Const. CV	0.11 (8)	0.22 (8)	0.25 (8)	0.05 (7)	0.23 (8)	0.2 (8)	≤ 0 (7)	0.06 (8)	0.13 (8)	≤ 0 (8)	≤ 0 (9)	0.15 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)
	Rand. CV	0.27 (7)	0.19 (8)	0.28 (8)	0.23 (7)	0.18 (7)	0.25 (8)	0.12 (7)	0.09 (7)	0.09 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.62 (3)	0.67 (3)	0.62 (3)	0.68 (3)	0.74 (3)	0.66 (3)	0.59 (3)	0.72 (3)	0.63 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	0.19 (3)	0.25 (3)	0.18 (3)	0.15 (3)	0.22 (3)	0.13 (3)	≤ 0 (3)	0.06 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	0.24 (3)	0.31 (3)	0.34 (3)	0.22 (3)	0.32 (3)	0.36 (3)	0.12 (3)	0.22 (3)	0.28 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.63 (2)	0.67 (2)	0.6 (2)	0.67 (2)	0.75 (2)	0.65 (2)	0.6 (2)	0.72 (2)	0.6 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.21 (2)	0.22 (2)	0.16 (2)	0.16 (2)	0.17 (2)	0.09 (2)	0.01 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.25 (2)	0.34 (2)	0.35 (2)	0.24 (2)	0.35 (2)	0.37 (2)	0.14 (2)	0.27 (2)	0.3 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 74: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.52 (7)	0.67 (8)	0.68 (8)	0.65 (7)	0.9 (8)	0.87 (8)	0.78 (7)	1.17 (8)	1.22 (8)	0.68 (8)	1.34 (8)	2.06 (8)	0.53 (7)	0.85 (8)	1.72 (9)	≤ 0 (8)	≤ 0 (8)	0.34 (8)
	Const. CV	≤ 0 (8)	0.19 (8)	0.25 (8)	≤ 0 (8)	0.12 (8)	0.14 (8)	≤ 0 (7)	0 (8)	≤ 0 (8)	≤ 0 (8)	0.09 (9)	0.34 (8)	≤ 0 (8)	≤ 0 (9)	0.5 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)
	Rand. CV	0.28 (7)	0.26 (7)	0.3 (8)	0.26 (7)	0.31 (8)	0.33 (7)	0.3 (7)	0.37 (7)	0.47 (8)	0.22 (8)	0.11 (8)	0.22 (8)	0.11 (7)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	0.68 (3)	0.71 (3)	0.65 (3)	0.87 (3)	0.97 (3)	0.9 (3)	1.1 (3)	1.29 (3)	1.25 (3)	1.04 (3)	1.45 (3)	1.83 (3)	0.87 (3)	0.98 (3)	1.32 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	0.02 (3)	0.2 (3)	0.16 (3)	≤ 0 (3)	0.11 (3)	0.07 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	0.31 (3)	0.35 (3)	0.35 (3)	0.35 (3)	0.52 (3)	0.51 (3)	0.34 (3)	0.58 (3)	0.65 (3)	0.3 (3)	0.37 (3)	0.41 (3)	0.12 (3)	0.06 (3)	0.03 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	0.7 (2)	0.71 (2)	0.64 (2)	0.93 (2)	0.99 (2)	0.89 (2)	1.15 (2)	1.34 (2)	1.26 (2)	1.2 (2)	1.54 (2)	1.65 (2)	1.05 (2)	0.83 (2)	0.9 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	0.05 (2)	0.17 (2)	0.14 (2)	≤ 0 (2)	0.05 (2)	0.03 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.04 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.34 (2)	0.37 (2)	0.37 (2)	0.38 (2)	0.51 (2)	0.54 (2)	0.38 (2)	0.62 (2)	0.68 (2)	0.34 (2)	0.39 (2)	0.37 (2)	0.15 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 75: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)
	Const. CV	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)
	Rand. CV	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 76: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, random costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (8)	0.28 (9)	1.14 (8)	≤ 0 (8)	≤ 0 (8)	0.68 (9)	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)	≤ 0 (8)	≤ 0 (8)	0.34 (8)
	Const. CV	≤ 0 (8)	0.05 (8)	0.64 (9)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)	≤ 0 (8)	0.06 (8)	0.4 (8)
	Rand. CV	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)	≤ 0 (8)
meanBase = 5	Uniform Demands	≤ 0 (3)	0.04 (3)	1.14 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Const. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)	≤ 0 (3)	≤ 0 (3)	≤ 0 (4)
	Rand. CV	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)	≤ 0 (3)
meanBase = 10	Uniform Demands	≤ 0 (2)	≤ 0 (2)	0.72 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Const. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)

Table 77: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.5`, random costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.29 (4)	0.46 (4)	0.52 (5)	0.29 (4)	0.41 (5)	0.52 (5)	0.23 (4)	0.36 (5)	0.47 (5)	≤ 0 (5)	≤ 0 (5)	0.6 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)
	Const. CV	0.36 (4)	0.57 (4)	0.7 (5)	0.34 (4)	0.63 (5)	0.69 (5)	0.26 (4)	0.59 (5)	0.62 (5)	≤ 0 (5)	≤ 0 (5)	0.24 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)
	Rand. CV	0.35 (4)	0.34 (5)	0.41 (5)	0.29 (4)	0.29 (4)	0.37 (4)	0.3 (4)	0.18 (4)	0.24 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)
meanBase = 5	Uniform Demands	0.44 (1)	0.5 (1)	0.46 (1)	0.46 (1)	0.53 (1)	0.52 (1)	0.35 (1)	0.44 (1)	0.39 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)
	Const. CV	0.54 (1)	0.61 (1)	0.59 (1)	0.54 (1)	0.64 (1)	0.58 (1)	0.41 (1)	0.51 (1)	0.45 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)
	Rand. CV	0.47 (1)	0.53 (1)	0.55 (1)	0.44 (1)	0.51 (1)	0.53 (1)	0.3 (1)	0.36 (1)	0.38 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.48 (1)	0.53 (1)	0.48 (1)	0.51 (1)	0.59 (1)	0.52 (1)	0.41 (1)	0.48 (1)	0.42 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)
	Const. CV	0.59 (1)	0.64 (1)	0.63 (1)	0.61 (1)	0.69 (1)	0.63 (1)	0.48 (1)	0.54 (1)	0.51 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)
	Rand. CV	0.53 (1)	0.58 (1)	0.62 (1)	0.52 (1)	0.57 (1)	0.62 (1)	0.37 (1)	0.42 (1)	0.43 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 78: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.11 (5)	0.36 (4)	0.43 (5)	≤ 0 (5)	0.19 (5)	0.41 (5)	≤ 0 (4)	0.14 (4)	0.22 (5)	≤ 0 (4)	0.56 (5)	2.1 (5)	≤ 0 (5)	0.61 (5)	2.11 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)
	Const. CV	0.29 (4)	0.53 (5)	0.59 (5)	0.26 (4)	0.61 (5)	0.67 (5)	0.16 (5)	0.59 (5)	0.56 (5)	≤ 0 (4)	0.52 (5)	1.57 (5)	≤ 0 (4)	0.21 (5)	1.3 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)
	Rand. CV	0.36 (4)	0.37 (5)	0.45 (5)	0.44 (4)	0.43 (5)	0.51 (4)	0.43 (4)	0.48 (5)	0.61 (4)	0.39 (4)	0.52 (4)	0.72 (5)	0.35 (4)	0.27 (5)	0.57 (4)	≤ 0 (5)	≤ 0 (5)	0.58 (5)
meanBase = 5	Uniform Demands	0.2 (1)	0.37 (1)	0.36 (1)	0.04 (1)	0.25 (1)	0.25 (1)	≤ 0 (1)	0.01 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	0.72 (2)	≤ 0 (1)	≤ 0 (2)	1.25 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)
	Const. CV	0.42 (1)	0.53 (1)	0.5 (1)	0.4 (1)	0.56 (1)	0.52 (2)	0.31 (1)	0.48 (1)	0.49 (2)	0.04 (1)	≤ 0 (2)	0.77 (2)	≤ 0 (1)	≤ 0 (2)	0.52 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)
	Rand. CV	0.47 (1)	0.55 (1)	0.57 (1)	0.53 (1)	0.64 (1)	0.66 (1)	0.6 (1)	0.73 (1)	0.76 (1)	0.56 (1)	0.64 (1)	0.83 (1)	0.46 (1)	0.39 (1)	0.37 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.21 (1)	0.4 (1)	0.38 (1)	0.05 (1)	0.27 (1)	0.27 (1)	≤ 0 (1)	0.03 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	0.71 (1)	≤ 0 (1)	≤ 0 (1)	1.24 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)
	Const. CV	0.46 (1)	0.57 (1)	0.53 (1)	0.44 (1)	0.62 (1)	0.57 (1)	0.36 (1)	0.54 (1)	0.54 (1)	0.06 (1)	≤ 0 (1)	0.86 (1)	≤ 0 (1)	≤ 0 (1)	0.5 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)
	Rand. CV	0.52 (1)	0.59 (1)	0.61 (1)	0.59 (1)	0.7 (1)	0.71 (1)	0.67 (1)	0.8 (1)	0.82 (1)	0.66 (1)	0.74 (1)	0.92 (1)	0.56 (1)	0.47 (1)	0.45 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 79: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (5)	0.48 (5)	1.41 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)
	Const. CV	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)
	Rand. CV	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)
meanBase = 5	Uniform Demands	≤ 0 (1)	≤ 0 (2)	1.76 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)
	Const. CV	≤ 0 (1)	≤ 0 (2)	1.47 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
meanBase = 10	Uniform Demands	≤ 0 (1)	≤ 0 (1)	1.07 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)
	Const. CV	≤ 0 (1)	≤ 0 (1)	0.93 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 80: Results for $L = 3$, $\ell = 2$, overPenalty = 0.75, uniform costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (5)	0.41 (5)	2.14 (5)	≤ 0 (5)	0.42 (5)	1.43 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)	≤ 0 (5)	0.48 (5)	1.41 (5)
	Const. CV	≤ 0 (5)	0.28 (5)	1.28 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)	≤ 0 (5)	0.44 (5)	0.89 (5)
	Rand. CV	≤ 0 (5)	≤ 0 (5)	0.5 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)	≤ 0 (5)	≤ 0 (5)	0.58 (5)
meanBase = 5	Uniform Demands	≤ 0 (1)	≤ 0 (2)	0.93 (2)	≤ 0 (1)	≤ 0 (2)	0.7 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)	≤ 0 (1)	≤ 0 (2)	1.76 (2)
	Const. CV	≤ 0 (1)	≤ 0 (2)	0.57 (2)	≤ 0 (1)	≤ 0 (2)	1.46 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)	≤ 0 (1)	≤ 0 (2)	1.47 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)
meanBase = 10	Uniform Demands	≤ 0 (1)	≤ 0 (1)	1.15 (1)	≤ 0 (1)	≤ 0 (1)	0.96 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)	≤ 0 (1)	≤ 0 (1)	1.07 (1)
	Const. CV	≤ 0 (1)	≤ 0 (1)	0.48 (1)	≤ 0 (1)	≤ 0 (1)	0.91 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)	≤ 0 (1)	≤ 0 (1)	0.93 (1)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 81: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, uniform costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.34 (5)	0.6 (5)	0.56 (5)	0.33 (4)	0.57 (6)	0.53 (5)	0.26 (4)	0.53 (5)	0.49 (5)	≤ 0 (6)	0.08 (6)	0.32 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)
	Const. CV	0.35 (5)	0.56 (5)	0.67 (5)	0.32 (4)	0.59 (6)	0.64 (5)	0.23 (4)	0.47 (5)	0.6 (5)	≤ 0 (6)	0.33 (6)	0.36 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)
	Rand. CV	0.2 (4)	0.04 (5)	0.09 (5)	0.14 (4)	≤ 0 (5)	0.01 (5)	0.04 (5)	≤ 0 (4)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (6)	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)
meanBase = 5	Uniform Demands	0.49 (1)	0.55 (1)	0.55 (2)	0.51 (1)	0.59 (1)	0.57 (2)	0.4 (1)	0.49 (1)	0.48 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)
	Const. CV	0.51 (1)	0.58 (1)	0.55 (2)	0.5 (1)	0.59 (1)	0.58 (2)	0.38 (1)	0.48 (1)	0.5 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)
	Rand. CV	0.2 (1)	0.13 (1)	0.09 (1)	0.18 (1)	0.07 (1)	0.03 (1)	0.05 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.54 (1)	0.58 (1)	0.53 (1)	0.57 (1)	0.65 (1)	0.56 (1)	0.47 (1)	0.56 (1)	0.47 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)
	Const. CV	0.56 (1)	0.63 (1)	0.58 (1)	0.57 (1)	0.66 (1)	0.6 (1)	0.46 (1)	0.58 (1)	0.5 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)
	Rand. CV	0.25 (1)	0.15 (1)	0.11 (1)	0.19 (1)	0.07 (1)	0.02 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 82: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.25 (5)	0.52 (5)	0.5 (5)	0.23 (5)	0.6 (5)	0.56 (6)	0.14 (5)	0.58 (5)	0.54 (6)	≤ 0 (5)	0.3 (6)	0.9 (6)	≤ 0 (5)	0.49 (5)	1.26 (6)	≤ 0 (6)	0.38 (6)	1.07 (5)
	Const. CV	0.39 (5)	0.63 (6)	0.68 (5)	0.49 (5)	0.76 (5)	0.89 (5)	0.6 (5)	0.97 (5)	1.11 (5)	0.45 (5)	1.91 (6)	2.82 (5)	0.29 (5)	1.45 (6)	2.11 (6)	≤ 0 (6)	0.67 (6)	0.99 (5)
	Rand. CV	0.04 (4)	≤ 0 (4)	≤ 0 (5)	0 (4)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	≤ 0 (5)	0 (5)	≤ 0 (6)	0.02 (6)	≤ 0 (4)	≤ 0 (5)	0.07 (6)	≤ 0 (5)	≤ 0 (6)	0.22 (5)
meanBase = 5	Uniform Demands	0.37 (1)	0.47 (1)	0.48 (2)	0.36 (1)	0.5 (1)	0.48 (2)	0.27 (1)	0.54 (2)	0.52 (2)	0.1 (2)	0.06 (2)	0.35 (2)	≤ 0 (2)	≤ 0 (2)	0.39 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)
	Const. CV	0.56 (1)	0.61 (1)	0.6 (2)	0.71 (1)	0.79 (1)	0.79 (2)	0.88 (1)	1.13 (2)	1.08 (2)	0.89 (2)	1.52 (2)	2.57 (2)	0.67 (2)	0.76 (2)	1.73 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)
	Rand. CV	≤ 0 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	0.04 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.4 (1)	0.49 (1)	0.46 (1)	0.41 (1)	0.56 (1)	0.49 (1)	0.31 (1)	0.52 (1)	0.48 (1)	0.04 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)
	Const. CV	0.61 (1)	0.66 (1)	0.61 (1)	0.78 (1)	0.88 (1)	0.81 (1)	0.98 (1)	1.17 (1)	1.11 (1)	0.96 (1)	1.19 (1)	2.12 (1)	0.72 (1)	0.48 (1)	1.27 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (1)	0.06 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 83: Results for $L = 3$, $\ell = 2$, overPenalty = 0.75, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)
	Const. CV	≤ 0 (6)	0.67 (6)	0.99 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)
	Rand. CV	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)
meanBase = 5	Uniform Demands	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)
	Const. CV	≤ 0 (2)	≤ 0 (2)	0.8 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)
	Const. CV	≤ 0 (1)	≤ 0 (1)	0.01 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 84: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, proportional costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		`chiBase` = 0			`chiBase` = 1			`chiBase` = 2			`chiBase` = 5			`chiBase` = 7			`chiBase` = 999		
	`CVBase` →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
`meanBase` = 1	Uniform Demands	≤ 0 (6)	0.29 (6)	1.06 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)	≤ 0 (6)	0.38 (6)	1.07 (5)
	Const. CV	≤ 0 (6)	0.8 (6)	2.05 (6)	≤ 0 (6)	0.66 (6)	1.06 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)	≤ 0 (6)	0.67 (6)	0.99 (5)
	Rand. CV	≤ 0 (5)	≤ 0 (6)	0.14 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)	≤ 0 (5)	≤ 0 (6)	0.22 (5)
`meanBase` = 5	Uniform Demands	≤ 0 (2)	≤ 0 (2)	0.51 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)	≤ 0 (2)	≤ 0 (2)	0.73 (2)
	Const. CV	≤ 0 (2)	≤ 0 (2)	1.42 (2)	≤ 0 (2)	≤ 0 (2)	0.72 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)	≤ 0 (2)	≤ 0 (2)	0.8 (2)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
`meanBase` = 10	Uniform Demands	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)	≤ 0 (1)	≤ 0 (1)	0.04 (2)
	Const. CV	≤ 0 (1)	≤ 0 (1)	1.04 (2)	≤ 0 (1)	≤ 0 (1)	0.07 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)	≤ 0 (1)	≤ 0 (1)	0.01 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 85: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, proportional costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.37 (6)	0.62 (7)	0.61 (7)	0.36 (6)	0.64 (6)	0.69 (7)	0.23 (6)	0.57 (7)	0.56 (7)	≤ 0 (6)	≤ 0 (7)	0.04 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)
	Const. CV	0.07 (6)	0.22 (7)	0.2 (7)	0.01 (6)	0.19 (6)	0.21 (7)	≤ 0 (7)	0.08 (7)	0.05 (7)	≤ 0 (6)	≤ 0 (7)	≤ 0 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)
	Rand. CV	0.21 (6)	0.16 (6)	0.21 (6)	0.17 (6)	0.13 (6)	0.17 (6)	0.08 (6)	0.04 (7)	0.08 (6)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	0.54 (2)	0.6 (2)	0.59 (2)	0.57 (2)	0.65 (2)	0.62 (2)	0.47 (2)	0.63 (2)	0.57 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)
	Const. CV	0.16 (2)	0.2 (2)	0.15 (2)	0.11 (2)	0.16 (2)	0.09 (2)	≤ 0 (2)	0.02 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)
	Rand. CV	0.2 (2)	0.3 (2)	0.34 (2)	0.17 (2)	0.27 (2)	0.35 (2)	0.08 (2)	0.2 (2)	0.27 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.59 (1)	0.61 (1)	0.57 (1)	0.62 (1)	0.68 (1)	0.61 (1)	0.53 (1)	0.59 (1)	0.53 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)
	Const. CV	0.19 (1)	0.23 (1)	0.14 (1)	0.15 (1)	0.18 (1)	0.07 (1)	≤ 0 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.24 (1)	0.3 (1)	0.33 (1)	0.22 (1)	0.31 (1)	0.34 (1)	0.11 (1)	0.2 (1)	0.23 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 86: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.46 (6)	0.64 (7)	0.64 (7)	0.58 (6)	0.79 (6)	0.84 (7)	0.7 (7)	1.1 (7)	1.06 (7)	0.58 (6)	1.7 (7)	2.44 (7)	0.42 (6)	1.25 (7)	2.07 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)
	Const. CV	≤ 0 (6)	0.19 (7)	0.2 (7)	≤ 0 (6)	0.09 (6)	0.17 (7)	≤ 0 (6)	≤ 0 (6)	0.05 (7)	≤ 0 (7)	0.02 (7)	0.36 (7)	≤ 0 (6)	0.17 (7)	1.04 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)
	Rand. CV	0.23 (6)	0.19 (6)	0.22 (6)	0.28 (6)	0.26 (6)	0.32 (6)	0.24 (6)	0.27 (6)	0.39 (7)	0.19 (6)	0.08 (7)	0.17 (7)	0.11 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	0.63 (2)	0.67 (2)	0.63 (2)	0.82 (2)	0.91 (2)	0.86 (2)	1.02 (2)	1.21 (2)	1.18 (2)	1.01 (2)	1.44 (2)	2.15 (3)	0.86 (2)	0.89 (2)	1.61 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)
	Const. CV	0.01 (2)	0.16 (2)	0.14 (2)	≤ 0 (2)	0.07 (2)	0.02 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	≤ 0 (3)	0.46 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)
	Rand. CV	0.29 (2)	0.33 (2)	0.34 (2)	0.32 (2)	0.48 (2)	0.49 (2)	0.3 (2)	0.5 (2)	0.6 (2)	0.25 (2)	0.26 (2)	0.25 (2)	0.1 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	0.65 (1)	0.67 (1)	0.61 (1)	0.86 (1)	0.9 (1)	0.84 (1)	1.07 (1)	1.23 (1)	1.18 (1)	1.09 (1)	1.38 (1)	1.96 (2)	0.93 (1)	0.78 (1)	1.53 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)
	Const. CV	0.04 (1)	0.18 (1)	0.12 (1)	≤ 0 (1)	0.08 (1)	0.03 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	0.31 (1)	0.34 (1)	0.33 (1)	0.34 (1)	0.46 (1)	0.47 (1)	0.34 (1)	0.54 (1)	0.6 (1)	0.29 (1)	0.29 (1)	0.27 (1)	0.11 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 87: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (6)	0.33 (7)	1.09 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)
	Const. CV	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)
	Rand. CV	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	≤ 0 (2)	≤ 0 (2)	0.06 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)
	Const. CV	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	≤ 0 (1)	≤ 0 (2)	0.06 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)
	Const. CV	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 88: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, random costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	≤ 0 (6)	0.48 (7)	2.06 (7)	≤ 0 (6)	0.3 (7)	1.01 (7)	≤ 0 (6)	0.33 (7)	1.06 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)	≤ 0 (6)	0.33 (7)	1.09 (7)
	Const. CV	≤ 0 (6)	0.41 (7)	0.94 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)	≤ 0 (6)	0.42 (7)	0.79 (7)
	Rand. CV	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)	≤ 0 (6)	≤ 0 (6)	≤ 0 (7)
meanBase = 5	Uniform Demands	≤ 0 (2)	≤ 0 (2)	1.33 (3)	≤ 0 (2)	≤ 0 (3)	≤ 0 (3)	≤ 0 (2)	≤ 0 (2)	0 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)	≤ 0 (2)	≤ 0 (2)	0.06 (3)
	Const. CV	≤ 0 (2)	≤ 0 (3)	0.56 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)	≤ 0 (2)	≤ 0 (3)	0.57 (3)
	Rand. CV	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)
meanBase = 10	Uniform Demands	≤ 0 (1)	≤ 0 (2)	1.26 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	0.02 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)	≤ 0 (1)	≤ 0 (2)	0.06 (2)
	Const. CV	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)

Table 89: Results for $L = 3$, $\ell = 2$, `overPenalty = 0.75`, random costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.22 (4)	0.46 (4)	0.46 (4)	0.2 (4)	0.46 (4)	0.54 (4)	0.15 (4)	0.42 (4)	0.47 (4)	≤ 0 (4)	0.09 (5)	0.8 (4)	≤ 0 (4)	0.99 (4)	2.05 (5)	2.18 (4)	5.27 (4)	7.65 (5)
	Const. CV	0.3 (4)	0.54 (4)	0.56 (4)	0.27 (4)	0.51 (4)	0.61 (4)	0.18 (4)	0.46 (4)	0.54 (4)	≤ 0 (4)	0.01 (5)	0.73 (4)	≤ 0 (4)	1.06 (4)	2.33 (4)	2.11 (4)	5.28 (4)	7.54 (5)
	Rand. CV	0.31 (3)	0.3 (4)	0.34 (4)	0.24 (4)	0.23 (4)	0.29 (4)	0.23 (3)	0.13 (4)	0.17 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	≤ 0 (4)	0.03 (4)	2.38 (4)	7.52 (4)	6.33 (4)	38.52 (4)
meanBase = 5	Uniform Demands	0.42 (1)	0.47 (1)	0.45 (1)	0.42 (1)	0.5 (1)	0.49 (1)	0.31 (1)	0.39 (1)	0.37 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	0.82 (2)	3.59 (2)	2.71 (1)	4.21 (2)	4.83 (2)
	Const. CV	0.52 (1)	0.58 (1)	0.57 (1)	0.51 (1)	0.6 (1)	0.54 (1)	0.37 (1)	0.47 (1)	0.41 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	0.68 (2)	3.42 (2)	2.57 (1)	3.96 (2)	4.64 (2)
	Rand. CV	0.43 (1)	0.5 (1)	0.52 (1)	0.4 (1)	0.47 (1)	0.49 (1)	0.26 (1)	0.33 (1)	0.34 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.16 (1)	19.52 (1)	24.89 (1)	24.77 (1)
meanBase = 10	Uniform Demands	0.46 (1)	0.49 (1)	0.44 (1)	0.47 (1)	0.52 (1)	0.46 (1)	0.34 (1)	0.39 (1)	0.33 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	1.02 (1)	4.18 (1)	3.45 (1)	5.22 (1)	5.44 (1)
	Const. CV	0.55 (1)	0.59 (1)	0.54 (1)	0.55 (1)	0.61 (1)	0.55 (1)	0.39 (1)	0.46 (1)	0.39 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.83 (1)	4.17 (1)	3.26 (1)	4.76 (1)	5.26 (1)
	Rand. CV	0.51 (1)	0.57 (1)	0.57 (1)	0.49 (1)	0.54 (1)	0.55 (1)	0.35 (1)	0.4 (1)	0.41 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.21 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)

Table 90: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, uniform costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.14 (3)	0.36 (4)	0.38 (4)	0.05 (3)	0.23 (4)	0.34 (4)	≤ 0 (3)	0.19 (4)	0.21 (4)	0.11 (3)	3 (4)	4.37 (4)	1.85 (4)	7.06 (4)	11.51 (4)	2.18 (4)	5.27 (4)	7.65 (5)
	Const. CV	0.24 (4)	0.43 (4)	0.53 (4)	0.22 (4)	0.5 (4)	0.59 (4)	0.2 (3)	0.48 (4)	0.61 (4)	0.36 (4)	3.65 (5)	4.98 (4)	4.48 (3)	7.07 (4)	11.76 (4)	2.11 (4)	5.28 (4)	7.54 (5)
	Rand. CV	0.33 (3)	0.34 (4)	0.37 (4)	0.37 (3)	0.39 (4)	0.43 (4)	0.38 (3)	0.43 (4)	0.49 (4)	0.35 (4)	0.74 (4)	1.56 (4)	0.6 (3)	2.45 (4)	5.3 (4)	7.52 (4)	6.33 (4)	38.52 (4)
meanBase = 5	Uniform Demands	0.18 (1)	0.35 (1)	0.35 (1)	0.03 (1)	0.23 (1)	0.26 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (1)	≤ 0 (1)	1.83 (2)	5.85 (2)	2.23 (1)	24.11 (2)	25.23 (2)	2.71 (1)	4.21 (2)	4.83 (2)
	Const. CV	0.39 (1)	0.51 (1)	0.5 (1)	0.39 (1)	0.54 (1)	0.56 (1)	0.3 (1)	0.49 (1)	0.49 (1)	0.42 (1)	4 (2)	8.19 (2)	4.74 (1)	25.61 (2)	23.3 (2)	2.57 (1)	3.96 (2)	4.64 (2)
	Rand. CV	0.44 (1)	0.53 (1)	0.54 (1)	0.51 (1)	0.61 (1)	0.63 (1)	0.57 (1)	0.7 (1)	0.72 (1)	0.54 (1)	0.81 (1)	1.46 (1)	0.55 (1)	3.76 (1)	6.73 (1)	19.52 (1)	24.89 (1)	24.77 (1)
meanBase = 10	Uniform Demands	0.19 (1)	0.36 (1)	0.34 (1)	≤ 0 (1)	0.22 (1)	0.22 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	2.08 (1)	6.28 (1)	2.24 (1)	27.21 (1)	28.06 (1)	3.45 (1)	5.22 (1)	5.44 (1)
	Const. CV	0.42 (1)	0.51 (1)	0.48 (1)	0.39 (1)	0.54 (1)	0.5 (1)	0.28 (1)	0.43 (1)	0.42 (1)	0.05 (1)	5.07 (1)	9.52 (1)	6.6 (1)	29.03 (1)	26.79 (1)	3.26 (1)	4.76 (1)	5.26 (1)
	Rand. CV	0.49 (1)	0.56 (1)	0.58 (1)	0.56 (1)	0.67 (1)	0.68 (1)	0.63 (1)	0.76 (1)	0.79 (1)	0.63 (1)	0.67 (1)	0.68 (1)	0.51 (1)	0.38 (1)	8.39 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)

Table 91: Results for $L = 3$, $\ell = 2$, overPenalty = 1, uniform costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	1.48 (4)	2.31 (4)	2.48 (5)	2.13 (4)	3.8 (4)	5.74 (5)	2.18 (4)	5.27 (4)	7.65 (5)	2.18 (4)	5.27 (4)	7.65 (5)	2.18 (4)	5.27 (4)	7.65 (5)	2.18 (4)	5.27 (4)	7.65 (5)
	Const. CV	1.43 (4)	2.26 (4)	2.85 (4)	2.06 (4)	3.83 (4)	5.49 (5)	2.11 (4)	5.28 (4)	7.54 (5)	2.11 (4)	5.28 (4)	7.54 (5)	2.11 (4)	5.28 (4)	7.54 (5)	2.11 (4)	5.28 (4)	7.54 (5)
	Rand. CV	1.28 (4)	1.62 (4)	3.12 (4)	1.17 (4)	2.13 (4)	6.86 (4)	4.51 (4)	4.71 (4)	11.01 (4)	7.52 (4)	6.33 (4)	38.52 (4)	7.52 (4)	6.33 (4)	38.52 (4)	7.52 (4)	6.33 (4)	38.52 (4)
meanBase = 5	Uniform Demands	2.71 (1)	4.21 (2)	4.48 (2)	2.71 (1)	4.21 (2)	4.83 (2)	2.71 (1)	4.21 (2)	4.83 (2)	2.71 (1)	4.21 (2)	4.83 (2)	2.71 (1)	4.21 (2)	4.83 (2)	2.71 (1)	4.21 (2)	4.83 (2)
	Const. CV	2.57 (1)	3.96 (2)	4.3 (2)	2.57 (1)	3.96 (2)	4.64 (2)	2.57 (1)	3.96 (2)	4.64 (2)	2.57 (1)	3.96 (2)	4.64 (2)	2.57 (1)	3.96 (2)	4.64 (2)	2.57 (1)	3.96 (2)	4.64 (2)
	Rand. CV	19.52 (1)	24.85 (1)	23.31 (1)	19.52 (1)	24.89 (1)	24.77 (1)	19.52 (1)	24.89 (1)	24.77 (1)	19.52 (1)	24.89 (1)	24.77 (1)	19.52 (1)	24.89 (1)	24.77 (1)	19.52 (1)	24.89 (1)	24.77 (1)
meanBase = 10	Uniform Demands	3.45 (1)	5.22 (1)	5.07 (1)	3.45 (1)	5.22 (1)	5.44 (1)	3.45 (1)	5.22 (1)	5.44 (1)	3.45 (1)	5.22 (1)	5.44 (1)	3.45 (1)	5.22 (1)	5.44 (1)	3.45 (1)	5.22 (1)	5.44 (1)
	Const. CV	3.26 (1)	4.76 (1)	5.03 (1)	3.26 (1)	4.76 (1)	5.26 (1)	3.26 (1)	4.76 (1)	5.26 (1)	3.26 (1)	4.76 (1)	5.26 (1)	3.26 (1)	4.76 (1)	5.26 (1)	3.26 (1)	4.76 (1)	5.26 (1)
	Rand. CV	≤ 0 (1)	30.25 (1)	28.24 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)	≤ 0 (1)	30.31 (1)	29.89 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)	≤ 0 (1)	30.31 (1)	29.89 (1)

Table 92: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, uniform costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	22.25 (4)	18.89 (5)	14.61 (4)	36.49 (4)	30.3 (4)	30.34 (4)	29.68 (4)	37.46 (4)	34.86 (5)	10.53 (4)	9.19 (4)	8.09 (5)	3.76 (4)	5.27 (4)	7.65 (5)	2.18 (4)	5.27 (4)	7.65 (5)
	Const. CV	17.54 (4)	16.14 (5)	14.15 (4)	27.1 (4)	23.24 (4)	26.61 (4)	16.66 (4)	25.19 (4)	21.72 (5)	2.33 (4)	5.28 (4)	7.54 (5)	2.11 (4)	5.28 (4)	7.54 (5)	2.11 (4)	5.28 (4)	7.54 (5)
	Rand. CV	21.32 (4)	24.52 (4)	20.97 (4)	25.41 (4)	28.15 (4)	28.56 (4)	36.02 (4)	32.92 (4)	30.76 (4)	24.69 (4)	27.18 (4)	38.15 (4)	21.6 (4)	24.09 (4)	38.52 (4)	7.52 (4)	6.33 (4)	38.52 (4)
meanBase = 5	Uniform Demands	18.4 (1)	26.38 (2)	25.53 (2)	7.85 (1)	12.53 (2)	14.21 (2)	2.93 (1)	4.61 (2)	5.45 (2)	2.71 (1)	4.21 (2)	4.83 (2)	2.71 (1)	4.21 (2)	4.83 (2)	2.71 (1)	4.21 (2)	4.83 (2)
	Const. CV	3.35 (1)	15.8 (2)	21.93 (2)	2.57 (1)	4.17 (2)	5.62 (2)	2.57 (1)	3.96 (2)	4.64 (2)	2.57 (1)	3.96 (2)	4.64 (2)	2.57 (1)	3.96 (2)	4.64 (2)	2.57 (1)	3.96 (2)	4.64 (2)
	Rand. CV	18.59 (1)	22.14 (1)	23.84 (1)	19.52 (1)	24.33 (1)	23.71 (1)	19.52 (1)	24.89 (1)	24.77 (1)	19.52 (1)	24.89 (1)	24.77 (1)	19.52 (1)	24.89 (1)	24.77 (1)	19.52 (1)	24.89 (1)	24.77 (1)
meanBase = 10	Uniform Demands	23.35 (1)	30.53 (1)	28.59 (1)	10.35 (1)	15.2 (1)	16.61 (1)	3.78 (1)	5.71 (1)	6.1 (1)	3.45 (1)	5.22 (1)	5.44 (1)	3.45 (1)	5.22 (1)	5.44 (1)	3.45 (1)	5.22 (1)	5.44 (1)
	Const. CV	4.39 (1)	18.98 (1)	24.91 (1)	3.26 (1)	5.05 (1)	6.51 (1)	3.26 (1)	4.76 (1)	5.26 (1)	3.26 (1)	4.76 (1)	5.26 (1)	3.26 (1)	4.76 (1)	5.26 (1)	3.26 (1)	4.76 (1)	5.26 (1)
	Rand. CV	≤ 0 (1)	27.09 (1)	28.83 (1)	≤ 0 (1)	29.6 (1)	28.7 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)	≤ 0 (1)	30.31 (1)	29.89 (1)	≤ 0 (1)	30.3 (1)	29.89 (1)

Table 93: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, uniform costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.38 (4)	0.46 (5)	0.64 (5)	0.28 (5)	0.53 (4)	0.61 (5)	0.21 (5)	0.35 (5)	0.56 (5)	≤ 0 (5)	0.1 (5)	0.68 (5)	≤ 0 (5)	0.83 (5)	1.91 (5)	58.47 (5)	46.17 (5)	42.83 (5)
	Const. CV	0.38 (4)	0.55 (5)	0.69 (5)	0.27 (5)	0.56 (4)	0.66 (5)	0.18 (5)	0.43 (5)	0.57 (5)	≤ 0 (5)	0.03 (4)	0.57 (5)	≤ 0 (5)	0.68 (5)	1.8 (5)	58.62 (5)	47.17 (5)	43.91 (5)
	Rand. CV	0.18 (5)	0.08 (4)	0.09 (5)	0.12 (5)	≤ 0 (5)	≤ 0 (5)	0.13 (4)	≤ 0 (5)	≤ 0 (4)	≤ 0 (4)	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	≤ 0 (5)	1.29 (5)	71.17 (4)	57.76 (5)	48.52 (5)
meanBase = 5	Uniform Demands	0.47 (1)	0.53 (1)	0.5 (1)	0.47 (1)	0.56 (1)	0.52 (1)	0.35 (1)	0.45 (1)	0.4 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	2.46 (2)	52.67 (1)	8.4 (2)	7.49 (2)
	Const. CV	0.48 (1)	0.55 (1)	0.54 (1)	0.46 (1)	0.55 (1)	0.54 (1)	0.34 (1)	0.43 (1)	0.42 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	2.37 (2)	53.51 (1)	9.25 (2)	8.03 (2)
	Rand. CV	0.19 (1)	0.09 (1)	0.07 (1)	0.13 (1)	0.04 (1)	≤ 0 (1)	0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	28.03 (2)	54.34 (1)	15.43 (2)
meanBase = 10	Uniform Demands	0.51 (1)	0.56 (1)	0.52 (1)	0.53 (1)	0.58 (1)	0.53 (1)	0.42 (1)	0.48 (1)	0.44 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	2.75 (1)	59.99 (1)	25.7 (1)	14.17 (1)
	Const. CV	0.53 (1)	0.61 (1)	0.58 (1)	0.52 (1)	0.62 (1)	0.58 (1)	0.41 (1)	0.53 (1)	0.48 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	2.57 (1)	60.76 (1)	27.19 (1)	15.28 (1)
	Rand. CV	0.22 (1)	0.12 (1)	0.06 (1)	0.16 (1)	0.03 (1)	≤ 0 (1)	0.01 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	72.84 (1)	61.53 (1)	54.39 (1)

Table 94: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, proportional costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.29 (4)	0.39 (5)	0.58 (5)	0.28 (4)	0.45 (4)	0.53 (5)	0.2 (4)	0.42 (5)	0.53 (5)	≤ 0 (5)	0.42 (4)	1.52 (5)	≤ 0 (4)	0.74 (5)	2.07 (5)	58.47 (5)	46.17 (5)	42.83 (5)
	Const. CV	0.4 (4)	0.56 (5)	0.7 (5)	0.49 (4)	0.73 (5)	0.76 (5)	0.56 (5)	0.92 (5)	1.09 (5)	0.6 (4)	3.17 (5)	4.88 (5)	0.4 (5)	3.71 (5)	5.25 (5)	58.62 (5)	47.17 (5)	43.91 (5)
	Rand. CV	0.03 (4)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (4)	0.04 (4)	≤ 0 (5)	≤ 0 (5)	0.16 (4)	0.23 (5)	0.92 (5)	0.34 (5)	3.07 (4)	5.97 (4)	71.17 (4)	57.76 (5)	48.52 (5)
meanBase = 5	Uniform Demands	0.34 (1)	0.45 (1)	0.43 (1)	0.34 (1)	0.47 (1)	0.44 (1)	0.27 (1)	0.41 (1)	0.4 (1)	0.03 (1)	≤ 0 (2)	0.63 (2)	≤ 0 (1)	≤ 0 (2)	0.88 (2)	52.67 (1)	8.4 (2)	7.49 (2)
	Const. CV	0.53 (1)	0.58 (1)	0.57 (1)	0.68 (1)	0.75 (1)	0.75 (1)	0.83 (1)	0.98 (1)	1.02 (2)	0.8 (1)	2.8 (2)	5.83 (2)	0.58 (1)	3.75 (2)	9.27 (2)	53.51 (1)	9.25 (2)	8.03 (2)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.03 (2)	0.2 (1)	0.61 (1)	0.49 (2)	3.37 (1)	6.61 (1)	28.03 (2)	54.34 (1)	15.43 (2)
meanBase = 10	Uniform Demands	0.38 (1)	0.47 (1)	0.45 (1)	0.37 (1)	0.51 (1)	0.48 (1)	0.31 (1)	0.49 (1)	0.44 (1)	0.04 (1)	≤ 0 (1)	0.55 (1)	≤ 0 (1)	≤ 0 (1)	0.77 (1)	59.99 (1)	25.7 (1)	14.17 (1)
	Const. CV	0.58 (1)	0.64 (1)	0.61 (1)	0.74 (1)	0.84 (1)	0.79 (1)	0.93 (1)	1.13 (1)	1.08 (1)	0.94 (1)	3.2 (1)	6.19 (1)	0.72 (1)	4.47 (1)	10.25 (1)	60.76 (1)	27.19 (1)	15.28 (1)
	Rand. CV	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.05 (1)	0.25 (1)	0.73 (1)	0.63 (1)	4.19 (1)	8.76 (1)	72.84 (1)	61.53 (1)	54.39 (1)

Table 95: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, proportional costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	2.68 (5)	2.04 (5)	2.18 (5)	4.77 (5)	4.15 (5)	5.41 (5)	5.01 (5)	7.41 (5)	9.91 (5)	8.14 (5)	10.26 (5)	13.17 (5)	9.43 (5)	12.13 (5)	16.54 (5)	58.47 (5)	46.17 (5)	42.83 (5)
	Const. CV	2.77 (5)	1.87 (5)	2.03 (5)	4.95 (5)	4.06 (5)	5.1 (5)	5.18 (5)	7.15 (5)	9.46 (5)	8.39 (5)	10.38 (5)	13.06 (5)	9.64 (5)	12.17 (5)	16.13 (5)	58.62 (5)	47.17 (5)	43.91 (5)
	Rand. CV	6.75 (4)	2.75 (5)	2.66 (5)	8.87 (4)	5.09 (5)	5.5 (5)	11 (4)	5.4 (5)	6.88 (5)	17.15 (4)	9.15 (5)	9.72 (5)	20.49 (4)	10.76 (5)	11.03 (5)	71.17 (4)	57.76 (5)	48.52 (5)
meanBase = 5	Uniform Demands	9.67 (1)	4.72 (2)	3.34 (2)	16.27 (1)	4.95 (2)	5.23 (2)	23.44 (1)	5.56 (2)	6.12 (2)	43.54 (1)	8.34 (2)	7.49 (2)	50.98 (1)	8.4 (2)	7.49 (2)	52.67 (1)	8.4 (2)	7.49 (2)
	Const. CV	9.07 (1)	4.27 (2)	3.14 (2)	15.28 (1)	4.91 (2)	5.15 (2)	22.04 (1)	5.55 (2)	6.04 (2)	41.59 (1)	9.07 (2)	8.03 (2)	50.26 (1)	9.25 (2)	8.03 (2)	53.51 (1)	9.25 (2)	8.03 (2)
	Rand. CV	5.84 (2)	9.57 (1)	4.91 (2)	6.41 (2)	18.57 (1)	5.39 (2)	7.12 (2)	28.23 (1)	6.3 (2)	10.84 (2)	50.32 (1)	13.49 (2)	13.83 (2)	54.08 (1)	15.41 (2)	28.03 (2)	54.34 (1)	15.43 (2)
meanBase = 10	Uniform Demands	12.61 (1)	6.06 (1)	3.86 (1)	20.75 (1)	14.83 (1)	10.39 (1)	29.21 (1)	21.26 (1)	12.98 (1)	50.96 (1)	25.65 (1)	14.17 (1)	58.36 (1)	25.7 (1)	14.17 (1)	59.99 (1)	25.7 (1)	14.17 (1)
	Const. CV	11.84 (1)	5 (1)	3.51 (1)	19.53 (1)	13.48 (1)	9.92 (1)	27.55 (1)	20.55 (1)	13.12 (1)	48.92 (1)	27.03 (1)	15.27 (1)	57.61 (1)	27.19 (1)	15.28 (1)	60.76 (1)	27.19 (1)	15.28 (1)
	Rand. CV	19.82 (1)	11.58 (1)	10.22 (1)	27.44 (1)	22.27 (1)	24.78 (1)	34.82 (1)	33.34 (1)	37.29 (1)	52.68 (1)	57.13 (1)	53.78 (1)	60.81 (1)	61.22 (1)	54.38 (1)	72.84 (1)	61.53 (1)	54.39 (1)

Table 96: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, proportional costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	1.84 (5)	1.8 (5)	2.22 (5)	3.78 (5)	3.6 (5)	4.81 (5)	3.59 (5)	6.53 (5)	8.9 (5)	7.55 (5)	9.95 (5)	12.72 (5)	9.95 (5)	11.63 (5)	15.63 (5)	58.47 (5)	46.17 (5)	42.83 (5)
	Const. CV	3.8 (5)	4.45 (4)	5.33 (5)	6.11 (5)	6.16 (5)	7.67 (5)	7.33 (5)	10.04 (5)	12.68 (5)	12.61 (5)	13.7 (5)	15.45 (5)	16.01 (5)	14.3 (5)	19.33 (5)	58.62 (5)	47.17 (5)	43.91 (5)
	Rand. CV	16.45 (4)	8.92 (5)	8.16 (5)	19.19 (4)	10.66 (5)	11.38 (5)	21.29 (4)	15.31 (5)	17.07 (5)	25.68 (4)	24.05 (5)	22.63 (5)	28.82 (4)	23.19 (5)	23.76 (5)	71.17 (4)	57.76 (5)	48.52 (5)
meanBase = 5	Uniform Demands	7.81 (1)	2.48 (2)	1.41 (2)	13.84 (1)	4.94 (2)	5.71 (2)	20.53 (1)	5.37 (2)	5.87 (2)	40.43 (1)	8.29 (2)	7.49 (2)	49.41 (1)	8.4 (2)	7.49 (2)	52.67 (1)	8.4 (2)	7.49 (2)
	Const. CV	15.36 (1)	8.74 (2)	9.85 (2)	23.17 (1)	8.81 (2)	10.21 (2)	31.07 (1)	6.96 (2)	7.62 (2)	49.57 (1)	9.25 (2)	8.03 (2)	53.23 (1)	9.25 (2)	8.03 (2)	53.51 (1)	9.25 (2)	8.03 (2)
	Rand. CV	16.58 (2)	33.62 (1)	18.3 (2)	12.59 (2)	45.51 (1)	13.84 (2)	15.04 (2)	52.17 (1)	13.43 (2)	23.62 (2)	54.34 (1)	15.43 (2)	27.99 (2)	54.34 (1)	15.43 (2)	28.03 (2)	54.34 (1)	15.43 (2)
meanBase = 10	Uniform Demands	10.26 (1)	3.1 (1)	1.5 (1)	17.8 (1)	13.19 (1)	9.92 (1)	25.83 (1)	20.01 (1)	12.6 (1)	47.77 (1)	25.6 (1)	14.17 (1)	56.83 (1)	25.7 (1)	14.17 (1)	59.99 (1)	25.7 (1)	14.17 (1)
	Const. CV	19.62 (1)	10.66 (1)	11.36 (1)	28.86 (1)	19.2 (1)	14.86 (1)	37.75 (1)	24.14 (1)	14.37 (1)	56.93 (1)	27.19 (1)	15.28 (1)	60.49 (1)	27.19 (1)	15.28 (1)	60.76 (1)	27.19 (1)	15.28 (1)
	Rand. CV	50.92 (1)	39.31 (1)	35.74 (1)	57.62 (1)	52.06 (1)	49.6 (1)	62.93 (1)	59.11 (1)	53.76 (1)	72.23 (1)	61.53 (1)	54.39 (1)	72.84 (1)	61.53 (1)	54.39 (1)	72.84 (1)	61.53 (1)	54.39 (1)

Table 97: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, proportional costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.34 (5)	0.64 (5)	0.59 (6)	0.31 (5)	0.6 (6)	0.58 (6)	0.2 (5)	0.49 (6)	0.54 (6)	≤ 0 (5)	0.13 (6)	0.22 (6)	≤ 0 (5)	0.64 (6)	1.68 (6)	77.08 (5)	61.71 (6)	52.43 (6)
	Const. CV	0.03 (5)	0.21 (6)	0.25 (6)	≤ 0 (5)	0.2 (6)	0.12 (6)	≤ 0 (5)	0.09 (6)	0.03 (6)	≤ 0 (5)	0.1 (6)	0.25 (6)	≤ 0 (5)	1.06 (6)	1.57 (6)	73.9 (5)	58.1 (6)	56.34 (6)
	Rand. CV	0.16 (5)	0.11 (5)	0.18 (5)	0.12 (5)	0.06 (5)	0.16 (6)	0.03 (5)	≤ 0 (5)	0.05 (5)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	≤ 0 (5)	≤ 0 (5)	≤ 0 (6)	76.17 (5)	74.14 (5)	67.76 (6)
meanBase = 5	Uniform Demands	0.52 (2)	0.61 (2)	0.55 (2)	0.53 (2)	0.65 (2)	0.62 (2)	0.43 (2)	0.6 (2)	0.53 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	0 (2)	2.16 (3)	69.67 (2)	54.13 (2)	24.99 (3)
	Const. CV	0.14 (2)	0.19 (2)	0.16 (2)	0.09 (2)	0.14 (2)	0.06 (2)	≤ 0 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (3)	≤ 0 (2)	0.15 (2)	2.32 (3)	66.18 (2)	50.58 (2)	24.54 (3)
	Rand. CV	0.19 (2)	0.27 (2)	0.32 (2)	0.16 (2)	0.27 (2)	0.33 (2)	0.03 (2)	0.17 (2)	0.26 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	71.97 (2)	66.83 (2)	63.28 (2)
meanBase = 10	Uniform Demands	0.56 (1)	0.61 (1)	0.54 (1)	0.59 (1)	0.66 (1)	0.56 (1)	0.49 (1)	0.58 (1)	0.46 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	1.62 (2)	54.53 (1)	16.13 (2)	12.64 (2)
	Const. CV	0.17 (1)	0.21 (1)	0.11 (1)	0.12 (1)	0.14 (1)	0.02 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (2)	≤ 0 (1)	≤ 0 (2)	1.19 (2)	50.87 (1)	15.35 (2)	12.13 (2)
	Rand. CV	0.22 (1)	0.28 (1)	0.3 (1)	0.19 (1)	0.27 (1)	0.3 (1)	0.08 (1)	0.16 (1)	0.18 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	61.88 (1)	52.29 (1)	45.96 (1)

Table 98: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, random costs and equal fractile capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	0.41 (6)	0.66 (5)	0.62 (6)	0.52 (6)	0.77 (6)	0.75 (6)	0.64 (5)	0.99 (6)	1.1 (6)	0.67 (6)	2.73 (6)	4.19 (6)	2.43 (5)	7.51 (6)	9.66 (6)	77.08 (5)	61.71 (6)	52.43 (6)
	Const. CV	≤ 0 (5)	0.18 (6)	0.24 (6)	≤ 0 (5)	0.13 (6)	0.07 (6)	≤ 0 (6)	0.07 (6)	0.14 (6)	0.07 (5)	1.35 (6)	2.1 (6)	0.59 (6)	4.17 (6)	5.53 (6)	73.9 (5)	58.1 (6)	56.34 (6)
	Rand. CV	0.19 (5)	0.14 (6)	0.2 (6)	0.23 (5)	0.19 (6)	0.27 (5)	0.19 (5)	0.19 (6)	0.35 (5)	0.14 (5)	0.53 (5)	0.73 (6)	0.63 (5)	1.62 (6)	3.27 (5)	76.17 (5)	74.14 (5)	67.76 (6)
meanBase = 5	Uniform Demands	0.6 (2)	0.66 (2)	0.61 (2)	0.78 (2)	0.88 (2)	0.82 (2)	0.98 (2)	1.18 (2)	1.17 (2)	1.1 (2)	2.91 (2)	5.07 (2)	3.17 (2)	15.35 (2)	19.4 (2)	69.67 (2)	54.13 (2)	24.99 (3)
	Const. CV	0 (2)	0.15 (2)	0.12 (2)	≤ 0 (2)	0.04 (2)	0.01 (2)	≤ 0 (2)	≤ 0 (2)	≤ 0 (2)	0.1 (2)	0.78 (2)	1.43 (2)	1.19 (2)	7.19 (2)	10.4 (3)	66.18 (2)	50.58 (2)	24.54 (3)
	Rand. CV	0.26 (2)	0.31 (2)	0.33 (2)	0.3 (2)	0.46 (2)	0.47 (2)	0.29 (2)	0.46 (2)	0.58 (2)	0.25 (2)	0.66 (2)	1.17 (2)	0.61 (2)	2.94 (2)	5.21 (2)	71.97 (2)	66.83 (2)	63.28 (2)
meanBase = 10	Uniform Demands	0.63 (1)	0.63 (1)	0.57 (1)	0.83 (1)	0.87 (1)	0.82 (1)	1.03 (1)	1.19 (1)	1.16 (1)	1.16 (1)	2.65 (1)	4.29 (2)	2.91 (1)	18.13 (1)	25.22 (2)	54.53 (1)	16.13 (2)	12.64 (2)
	Const. CV	0.03 (1)	0.17 (1)	0.09 (1)	≤ 0 (1)	0.05 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	≤ 0 (1)	0.41 (1)	1.13 (2)	1.13 (1)	6.69 (1)	13.25 (2)	50.87 (1)	15.35 (2)	12.13 (2)
	Rand. CV	0.29 (1)	0.31 (1)	0.31 (1)	0.33 (1)	0.43 (1)	0.44 (1)	0.33 (1)	0.49 (1)	0.56 (1)	0.29 (1)	0.59 (1)	0.99 (1)	0.6 (1)	3.63 (1)	6.08 (1)	61.88 (1)	52.29 (1)	45.96 (1)

Table 99: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, random costs and random capacities. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	11.78 (5)	2.83 (6)	2.05 (6)	25.95 (5)	7.43 (6)	8.09 (6)	27.05 (5)	14.03 (6)	16.17 (6)	38.67 (5)	48.25 (6)	52.38 (6)	45 (5)	61.71 (6)	52.43 (6)	77.08 (5)	61.71 (6)	52.43 (6)
	Const. CV	12.68 (5)	3.09 (6)	1.98 (6)	28.68 (5)	8.33 (6)	7.77 (6)	22.77 (5)	15.31 (6)	16.93 (6)	44.1 (5)	56.4 (6)	56.34 (6)	52.92 (5)	58.1 (6)	56.34 (6)	73.9 (5)	58.1 (6)	56.34 (6)
	Rand. CV	18.47 (5)	12.58 (5)	5.34 (6)	32.28 (5)	29.12 (5)	8.94 (6)	35.47 (5)	23.87 (5)	12.19 (6)	45.89 (5)	39.68 (5)	35.44 (6)	42.19 (5)	58.96 (5)	67.53 (6)	76.17 (5)	74.14 (5)	67.76 (6)
meanBase = 5	Uniform Demands	36.53 (2)	13.31 (2)	3.73 (3)	49.19 (2)	32.64 (2)	14.59 (3)	68.5 (2)	54.09 (2)	24.92 (3)	69.67 (2)	54.13 (2)	24.99 (3)	69.67 (2)	54.13 (2)	24.99 (3)	69.67 (2)	54.13 (2)	24.99 (3)
	Const. CV	33.49 (2)	13.44 (2)	3.57 (3)	58.01 (2)	38.78 (2)	14.84 (3)	66.18 (2)	50.58 (2)	24.54 (3)	66.18 (2)	50.58 (2)	24.54 (3)	66.18 (2)	50.58 (2)	24.54 (3)	66.18 (2)	50.58 (2)	24.54 (3)
	Rand. CV	46.5 (2)	34.51 (2)	29.23 (2)	65.99 (2)	58.82 (2)	52.97 (2)	71.96 (2)	66.82 (2)	63.27 (2)	71.97 (2)	66.83 (2)	63.28 (2)	71.97 (2)	66.83 (2)	63.28 (2)	71.97 (2)	66.83 (2)	63.28 (2)
meanBase = 10	Uniform Demands	54.53 (1)	9.79 (2)	4.03 (2)	54.53 (1)	16.13 (2)	12.55 (2)	54.53 (1)	16.13 (2)	12.64 (2)	54.53 (1)	16.13 (2)	12.64 (2)	54.53 (1)	16.13 (2)	12.64 (2)	54.53 (1)	16.13 (2)	12.64 (2)
	Const. CV	50.87 (1)	9.64 (2)	3.6 (2)	50.87 (1)	15.35 (2)	12.1 (2)	50.87 (1)	15.35 (2)	12.13 (2)	50.87 (1)	15.35 (2)	12.13 (2)	50.87 (1)	15.35 (2)	12.13 (2)	50.87 (1)	15.35 (2)	12.13 (2)
	Rand. CV	61.88 (1)	52.28 (1)	45.89 (1)	61.88 (1)	52.29 (1)	45.96 (1)	61.88 (1)	52.29 (1)	45.96 (1)	61.88 (1)	52.29 (1)	45.96 (1)	61.88 (1)	52.29 (1)	45.96 (1)	61.88 (1)	52.29 (1)	45.96 (1)

Table 100: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, random costs and equal fractile capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.

		chiBase = 0			chiBase = 1			chiBase = 2			chiBase = 5			chiBase = 7			chiBase = 999		
	CVBase →	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4	0.15	0.3	0.4
meanBase = 1	Uniform Demands	30.42 (5)	15.42 (6)	10.47 (6)	29.1 (5)	25.35 (6)	26.73 (6)	41.4 (5)	35.7 (6)	40.57 (6)	49.46 (5)	73.07 (6)	66.54 (6)	68.62 (5)	69.73 (6)	56.03 (6)	77.08 (5)	61.71 (6)	52.43 (6)
	Const. CV	24.03 (5)	8.75 (6)	6.81 (6)	25.95 (5)	16.53 (6)	16.57 (6)	35.41 (5)	24.4 (6)	26.85 (6)	38.61 (5)	63.02 (6)	56.44 (6)	67.46 (5)	58.11 (6)	56.34 (6)	73.9 (5)	58.1 (6)	56.34 (6)
	Rand. CV	27.69 (5)	21.44 (5)	11.64 (6)	36.16 (5)	26.65 (5)	15.62 (6)	27.67 (5)	31.98 (5)	20.11 (6)	32.48 (5)	38.59 (5)	51.2 (6)	35.26 (5)	65.99 (5)	70.16 (6)	76.17 (5)	74.14 (5)	67.76 (6)
meanBase = 5	Uniform Demands	56.67 (2)	30.62 (2)	21.73 (3)	71.98 (2)	59.16 (2)	51.41 (3)	71.81 (2)	57.63 (2)	43.16 (3)	69.93 (2)	54.18 (2)	24.98 (3)	69.68 (2)	54.13 (2)	24.99 (3)	69.67 (2)	54.13 (2)	24.99 (3)
	Const. CV	46.22 (2)	19.7 (2)	12.76 (3)	66.5 (2)	50.39 (2)	34.48 (3)	66.33 (2)	50.81 (2)	25.67 (3)	66.18 (2)	50.58 (2)	24.54 (3)	66.18 (2)	50.58 (2)	24.54 (3)	66.18 (2)	50.58 (2)	24.54 (3)
	Rand. CV	60.03 (2)	44.12 (2)	36.17 (2)	72.03 (2)	66.26 (2)	61.84 (2)	72.24 (2)	67.12 (2)	63.61 (2)	71.97 (2)	66.83 (2)	63.28 (2)	71.97 (2)	66.83 (2)	63.28 (2)	71.97 (2)	66.83 (2)	63.28 (2)
meanBase = 10	Uniform Demands	59.13 (1)	43.08 (1)	30.42 (2)	56.69 (1)	36.24 (1)	26.76 (2)	55.13 (1)	20.98 (2)	16 (2)	54.53 (1)	16.13 (2)	12.64 (2)	54.53 (1)	16.13 (2)	12.64 (2)	54.53 (1)	16.13 (2)	12.64 (2)
	Const. CV	51.05 (1)	27.02 (2)	17.68 (2)	50.88 (1)	16.29 (2)	13.81 (2)	50.87 (1)	15.35 (2)	12.13 (2)	50.87 (1)	15.35 (2)	12.13 (2)	50.87 (1)	15.35 (2)	12.13 (2)	50.87 (1)	15.35 (2)	12.13 (2)
	Rand. CV	61.93 (1)	53.58 (1)	49.01 (1)	61.88 (1)	52.36 (1)	46.24 (1)	61.88 (1)	52.27 (1)	45.94 (1)	61.88 (1)	52.29 (1)	45.96 (1)	61.88 (1)	52.29 (1)	45.96 (1)	61.88 (1)	52.29 (1)	45.96 (1)

Table 101: Results for $L = 3$, $\ell = 2$, `overPenalty = 1`, random costs and random capacities based on 4-period demands. Some optimality gaps were slightly smaller than zero due to variability in the Monte Carlo simulations; these are denoted by '≤ 0'. Cycle lengths provided in parentheses.