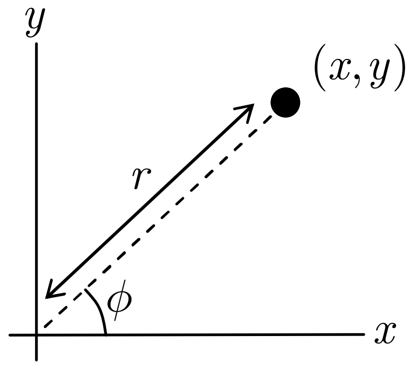
Coordinate Systems

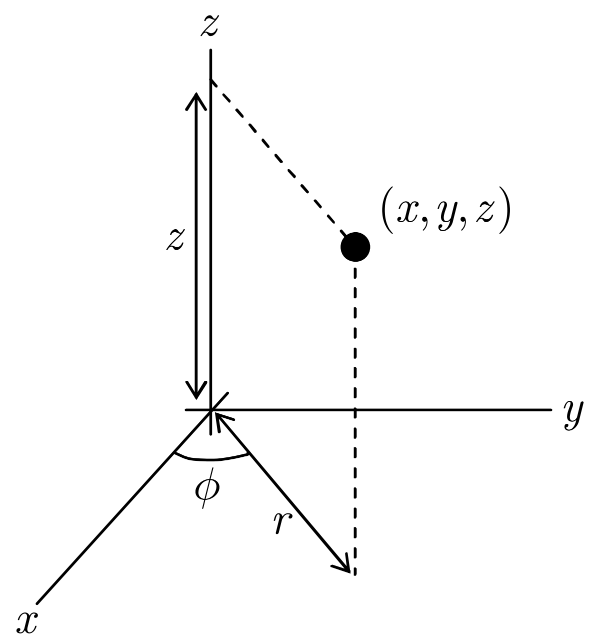
- Plane polar coordinates (2-Dimensions)



The position of the point can be specified either by (x,y), or by $\langle r,\phi \rangle$.

- $x = r\cos\phi$
- $y = r\sin\phi$
- $r = \sqrt{x^2 + y^2}$ $\qquad (r \geq 0)$
- $\phi = \tan^{-1}\dfrac{y}{x}$ $\qquad (0 \leq \phi < 2\pi)$
- The area element is given by $\mathrm{d}A = r\,\mathrm{d}r\,\mathrm{d}\phi$

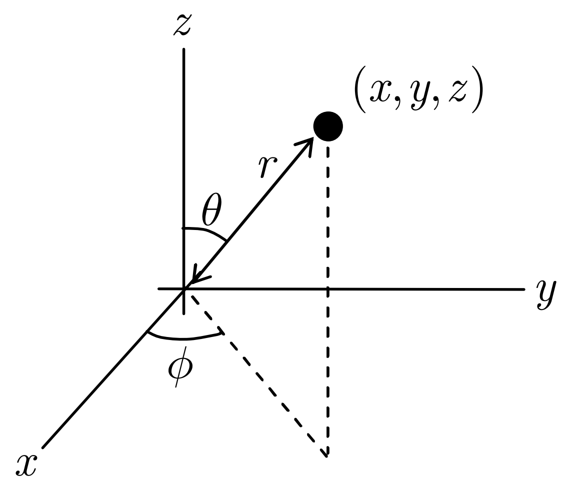
- Cylindrical Polar Coordinates



The position of the point can be specified either by (x,y,z), or by $\langle r,\phi,z \rangle$.

- $x = r\cos\phi$
- $y = r\sin\phi$
- $z = z$
- $r = \sqrt{x^2 + y^2}$ $\qquad (r \geq 0)$
- $\phi = \tan^{-1}\dfrac{y}{x}$ $\qquad (0 \leq \phi < 2\pi)$
- The volume element is given by $\mathrm{d}V = r\,\mathrm{d}r\,\mathrm{d}\phi\,\mathrm{d}z$

- Spherical Polar Coordinates



The position of the point can be specified either by (x, y, z), or by $\langle r, \theta, \phi \rangle$ (note, however, that various conventions exist as to the order).

- $x = r\sin\theta\cos\phi$
- $y = r\sin\theta\sin\phi$
- $z = r\cos\theta$
- $r = \sqrt{x^2 + y^2 + z^2}$ $\quad (r \geq 0)$
- $\phi = \tan^{-1}\dfrac{y}{x}$ $\quad (0 \leq \phi < 2\pi)$
- $\theta = \cos^{-1}\dfrac{z}{r} = \cos^{-1}\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$ $\quad \boxed{(0 \leq \phi \leq \pi)}$
- The volume element is given by $\mathrm{d}V = r^2 \sin\theta \,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi$

Notes:

- θ is the **polar angle**. ϕ is the **azimuthal angle**.
- The ***x*-*z*** plane is the **meridinal plane**, while the ***x*-*y*** plane is the **equatorial plane**.

