Elementary Analysis – Continuity and Differentiability

- **Continuity** – there are three good definitions of continuity

> *f(x)* is continuous at the point x_0 if and only if
> $$\lim_{\delta \to 0^+} f(x_0 + \delta) = \lim_{\delta \to 0^+} f(x_0 - \delta)$$

> *f(x)* is continuous at the point x_0 if and only if
> $$\lim_{x \to x_0} f(x) = f(x_0)$$
> As x approaches x_0 from both directions.

> *f(x)* is continuous at x_0 if and only if for every $\varepsilon > 0$, there exists δ such that
> $$|f(x_0) - f(x)| < \varepsilon \quad \text{for any } |x - x_0| < \delta$$

- **Differentiability** – again, there are several good definitions of differentiability – one of them is:

> *f(x)* is differentiable at the point x_0 if and only if the limit
> $$\lim_{\delta \to 0} \frac{f(x_0 + \delta) - f(x_0)}{\delta}$$
> Exists as δ approaches 0 from both directions.

To prove differentiability, it's good enough to differentiate the function and then find the values at which the derivative has discontinuities. The function is not differentiable there.